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Weak bus-constrained PMU placement for complete observability of a connected power network considering voltage stability indices

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Abstract

Phasor measurement units (PMUs) are preferred for installation at weak buses in a power network. Therefore, the weak buses need to be located and the strategic locations of PMUs identified to ensure network observability. Thus, the primary aim of this work is to identify the placements of the maximum number of PMUs installed at the weak buses in the electrical network. The voltage collapse proximity indicator, line stability index, fast voltage stability index, and a new voltage stability indicator utilizing load flow measurement are used to determine the weak buses. A novel deterministic methodology based on a binary-integer linear programming model is then proposed to determine the optimal locations of PMUs. The effect of a single PMU outage considering the weak buses is also demonstrated. The effectiveness of the developed approach is tested and validated on the standard IEEE 14-, 118-, 300-, and New England 39-bus systems. The obtained results are also compared to those using different weak bus methodologies.

Keywords: Binary integer linear programming, Optimal PMU placement, Observability, Phasor measurement unit, Voltage stability indices

1 Introduction

Power grid failure can occur because of a lack of connected power network infrastructure, incompetent asset maintenance, or increase in electricity consumption that aggravates power generation, transmission, and distribution. Such failures have resulted in losses of billions of dollars for the electric power industry, and are a concern to customers. State estimation of the connected power network is necessary to attain its effective interoperability, higher scalability, and stability [1]. These operations have usually been performed by supervisory control and data acquisition (SCADA) systems [2]. However, SCADA gives unsynchronized measurements which could easily lead to erroneous estimation of connected power network statuses [3, 4]. In addition, the data test rate is about 2–4 samples per cycle, which is very low and makes it indecisive about capturing minimal disruptions of the order of sub-seconds on the power network. These problems can be eliminated by installing PMUs in the connected power network [5].

PMUs are becoming very important to electrical engineers and researchers as they can furnish synchronized sub-second measurements of real-time voltage and current phasors via GPS technology [6], for complex power network monitoring, preservation, and control operations [7]. They are widely placed in various utilities for postmortem investigation, adaptive preservation, network preservation design, and state estimation [8, 9]. By using PMU measurements, the efficacy and robustness of power networks can be enhanced [10].

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One of the significant problems when developing PMU technology is the optimal PMU location. Currently, the installations of PMU have been slow because of the high investment required for the placement sites [11]. The PMU placement sites need to have suitable communication infrastructures for the PMUs to operate and this has also limited PMU installation. Furthermore, the cost of the PMU is expensive though it is expected to be reduced with increased demand in the future [11]. Hence, the PMU placement problem needs to be addressed. The primary aim of this work is to unravel the optimal PMU placement (OPP) problem while considering weak bus constraints across different IEEE standard systems.

Recently, several approaches have been proposed using various optimization techniques for the OPP issue. These can be classified as deterministic and stochastic algorithms [12, 13]. Different deterministic programming approaches have been used. In [14], the ILP-based formulation to determine the OPP problem is proposed where linear constraints are developed. The method is based on a binary bus-to-bus connection matrix which makes the evaluation of power network observability simple. However, the developed constraints become nonlinear when the existence of a zero injection bus is considered. In [15], a similar ILP approach that considers both conventional measurement and zero injection bus is proposed, which also eliminates the nonlinear constraints using a permutation matrix. However, the static node selection or static branch-and-bound algorithm (BBA) based ILP has a particular rule for choosing the next sub-problem in the procedure [16].

The static BBA method uses only one process for identification of the search order to build the inherent tree structure. This leads to a single optimal solution even when there are multiple sets of solutions available. The proposed method is extended in [17] by considering the cases of complete and incomplete observability such as the depth-of-one unobservability (DOU1) and depth-of-two unobservability (DOU2) concepts described in [18–21]. The proposed method is further extended by including the outage of a PMU. In order to obtain the best PMU employment set, a bus observability index and system observability redundancy index (SORI) are described. In [22], an integrated process is developed to unravel the minimal number and strategic locations of PMUs to make the power network fully monitorable and thereby able to be used for state estimation. The OPP issue is solved as a BILP, in which the discrete values $\{0, 1\}$ find the optimal location for a PMU. In [23], a two-step technique is established to unravel the solution for the OPP issue to attain full observability of large networks. The algorithmic procedure uses a binary search approach to partition the tree into sub-trees. This technique separates the spanning-tree using the ILP, while the ILP is computed dependent on the vertex-to-vertex connection information of the connected power network. After disintegration, PMUs are installed strategically in the subnetworks to minimize their placement cost.

A MILP is proposed in [24, 25] to find effective solutions for the OPP issue considering the effects of the zero injection measurements, including line and PMU outages. However, the presented solution is only feasible rather than optimal. In [26], a novel equivalent ILP approach for the PMU placement issue is proposed, though only a single solution set is obtained as the PMU placement issue is a combinatorial optimization issue. In [27], a novel methodology for the OPP problem is developed to ensure connected power network observability considering zero injection bus. The binary-ILP based approach is presented in [28] to attain full monitorability of the network while maximizing the redundancy measurement. The issue of the OPP and power flow measurements to attain observability under power grid fault conditions using binary-ILP is suggested in [29]. A three-step OPP problem-based approach is shown in [30], in which PMUs are initially placed on every bus of the connected power network while inessential bus locations are then determined and the most promising buses to retain PMUs with tree-based pruning applications are discussed. In [31], the OPP methodology for state estimation is proposed. This depends on vertex-to-vertex connection information considering single PMU and line outage. However, the solution provided has significant limitations.

An integer quadratic programming technique is proposed in [32] to find the most promising PMU locations while maximizing the redundancy measurement. An ILP methodology is proposed in [33] for power network observability considering the OPP problem while stipulating a fixed communication channel for PMUs. An integer programming method for identification of the OPP problem for full power network observability is proposed in [34], and a multi-criteria decision-making technique is explored to find the most promising sites. In [35], the proposed sequential quadratic programming (SQP) provides multiple sets of optimum solutions for the OPP issue, while the optimal solution set having the maximum SORI value indicates the best possible solution. Another work in [36] uses a binary semi-definite programming (BSDP) approach for the determination of the OPP issue. This intends to provide the minimal number and strategic locations of PMUs. However, although the results obtained from the SQP and BSDP methods are better than some earlier works, in some cases the results are not feasible. For example, the results are not feasible for the IEEE 30-, 57-, and 118-bus systems with zero injection buses using the SQP approach. Again, the results considering zero injection buses are not feasible for the IEEE 57-bus using BSDP. This is probably because of the numerical complexity developed in these methods, which complexity makes the formed constraints inaccurate for dettermining the OPP problem. Also, in [37], linear and nonlinear models are developed to solve the OPP problem, and the multiple solutions with the nonlinear model are shown and the solutions are ranked with a redundancy measurement index. A linear programming-based BBA approach is proposed by Theodorakatos in [38] for identification of the OPP problem to achieve full monitorability of the network, while redundancy measurement is also proposed to rank the solutions.

Many methods and techniques have been proposed to address the PMU placement problem considering various contingencies. However, most works deal only with the OPP problem while overlooking the bus weakness. Voltage collapse is a significant issue that affects network operating security [39]. Therefore, a precise estimation of voltage security is needed to identify the post-contingency voltage issues to avoid blackout. The early discovery of voltage instability from the network statuses given by synchrophasor quantities is addressed by Glavic and Cutsem [40]. Reference [41] proposes contingency-constrained oriented installation of PMUs for complete connected power network observability.

In [42], an artificial neural network-based approach is proposed for measuring the voltage stability margin under intact conditions as well as contingencies. Also, in [43], a novel voltage instability indicator that depends on phasor measurements at high sampling rate is proposed, while the efficacy and robustness of the indicator are examined at an extra-high voltage load and “transit” buses. In [44], it is observed that severe variations of the connected power network condition due to sudden changes of generation and load, or branch trip, affect system frequency, voltage, and current phasor. Thus, for effective online monitoring, high resolution PMU should be used. In [45, 46], two most basic wide-area voltage protection methods are proposed which are competent to face the security and stability problems in the connected power network.

In order to make the best use of PMUs, it has been proposed to place them at the weakest buses in a connected power network. The weakest bus refers to a bus which has the maximum voltage phasor change due to system variations [47]. The system variations can be load changes, faults, or contingencies in a connected power network. Hence, installing PMUs on the weakest buses in a connected power network can immediately reflect the voltage phasor variations on the weakest buses during changes in the connected power network. By investigating the PMU information on the weakest buses, the power network operators will be able to predict the power network condition and to take proper actions to avoid blackout.

In this work, different voltage stability indices are proposed to find the weakest buses and determine the strategic locations of PMUs. Most previous methods for solving the OPP problem only consider the issue of minimizing the number of PMUs to make the complete power network observable but ignore the vulnerability of the weakest buses. Although some previous studies have presented the methods of OPP problem considering the weakest buses, two main drawbacks exist in these techniques. The first is that the methods are not scalable, while the other is that in these methods, an OPP without considering the weakest buses is solved first and the placement considering the weakest buses is then obtained by amending the previous solution. Thus, the procedure of minimizing the number of PMUs and installing fewer PMUs on the weakest buses become two distinct processes that may reduce the range of optimal solutions.

Thus, the novelty of this work lies in the proposal of simultaneously running the processes of minimizing the number of PMUs and installing fewer PMUs on the weakest buses, using binary integer linear programming. The main contributions of this work are as follows:

i. To identify the weak buses, voltage stability indices are considered such as the voltage collapse proximity indicator (VCPI), line stability index (LSI), fast voltage stability index (FVSI), and new voltage stability indicator (NVSI) using the power flow from MATPOWER [48].
ii. To solve the OPP problem using binary integer linear programming and to ensure that all weak buses are directly observable.
iii. To test the developed methodology on the IEEE 14- [49], 118- [49], 300- [49], New England 39-bus [50] test systems and examine the results.

The rest of the paper is organized as follows. Topological observability analysis based on PMUs is described in Section 2 while Section 3 clarifies the weak bus measurement methodology. In Section 4, weak bus-constrained binary integer linear programming methods are described. Case studies and detailed results are discussed in Sections 5 and 6 draws the conclusion.

2 Topological observability analysis based on PMU

A PMU fitted at a specific bus can accurately determine the bus phase voltage and current phasor of every line connected to it considering channel capacity in electrical networks. Accordingly, an electrical network is said to be fully observable when all of its states are directly or indirectly measured. There are two ways to attain power network observability: the numerical method or the topological method [15, 17, 32, 51, 52]. The numerical approach is competent in systematically installing PMUs in power networks where there are hardly any actual measurements. However, the topology-based approach is easier to use in power networks that may be unobservable and have some observable islands [14, 53, 54]. Hence, in this work, the topological approach is used to attain the observability of the network as depicted in Fig. 1 [55].

As shown in Fig. 1a, with the installation of a PMU at bus 3, all the contiguous buses 1, 2, and 4 become observable. A zero injection bus (ZIB) is a bus that is not connected to either generator or load. By connecting ZIB with all observable buses other than one, the observation of an unobservable bus may be achieved by applying the KCL at ZIB. As shown in Fig. 1b, bus 3 is a ZIB that is connected by buses 1, 2, 4 and thus, buses 2, 3 and 4 are observed by PMU except bus 1. Hence, to attain the observability for bus 1, KCL is used. However, if an unobservable ZIB is connected by all the observable buses the unobserved ZIB may be made observable by applying KCL. As depicted in Fig. 1c, consider that the unobservable bus 3 is a ZIB and is connected by all the observable buses (1, 2, and 4), and the unobserved ZIB is made observable using KCL. If a ZIB is connected to more than two radial buses (RB), the PMU must be installed at the ZIB for the observation of RB attached to it. Considering a PMU placed at the ZIB bus 3, as shown in Fig. 1d, which is connected with all the radial buses (1, 2 and 4), the PMU at bus 3 not only observes that bus but all the buses connected to it. Thus, PMUs at the radial buses are avoided for cost consideration and the fact that they would only make one bus observable. As portrayed by Fig. 1e, if a PMU is installed at RB bus 3, it observes only bus 2 and itself.

Figure 2 shows the ZIB in a power system network. In Fig. 2a, a PMU is installed on bus 1, while bus 2 is a ZIB and bus 3 is a load bus. Bus 1 is directly observable while bus 2 is connected to the PMU bus and is also observable by solving the voltage equation considering voltage drop. As bus 2 is a ZIB, the current flowing over lines 1–2 and 2–3 are equal. By knowing the voltage at bus 2 and the current phasor on line 2–3, the voltage on bus 3 can be computed using Ohm's law. As a result, buses 1, 2, and 3 are all observable when bus 2 appears to be a ZIB. However, if bus 2 is not a ZIB, the current phasors on lines 1–2 and 2–3 are not equal. Thus, voltage at bus 3 cannot be calculated and only bus 1 and 2 are observable. Therefore, consideration of ZIB minimizes the number of PMUs required to make the power network completely observable. Figure 2b shows the influence of ZIB. As bus 3 is the ZIB, using KCL at bus 3 yields:

$$I_{13} + I_{23} + I_{43} = 0 \tag{1}$$

3 Weak bus measurement method

Voltage collapse is a system instability usually associated with the shortage of reactive power at the load end involving the entire power system [56]. The most effective method to counter voltage collapse is to identify the possible branches or buses where voltage

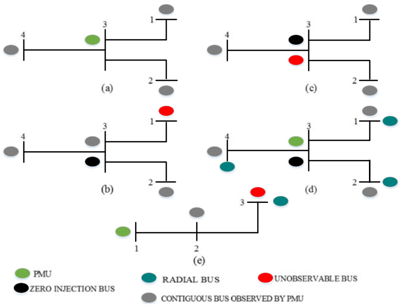

Fig. 1 PMU based topological observability of the power network

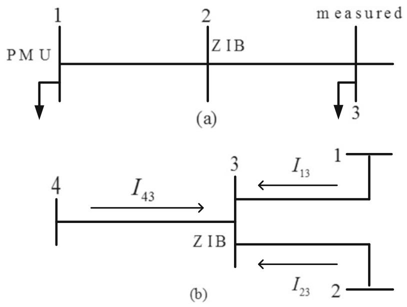

Fig. 2 ZIB in the power system network

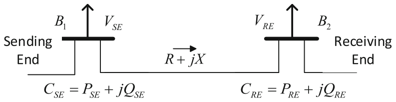

Fig. 3 Single line diagram for the 2-bus system

collapse may occur, i.e., to find the weakest buses in the system [57].

These methods are based on the concept of power flow through a single line, and different voltage stability indices are used to identify the weak buses. The single line diagram of an interconnected two-bus system network is shown in Fig. 3.

where,

C_{SE}, C_{RE}-Complex power at sending and receiving bus; V_{SE}, V_{RE}-Voltage magnitude at sending and receiving bus; δ_{SE}, δ_{RE}- Phase angles at sending and receiving bus; P_{SE}, Q_{SE}- Active and reactive power at sending bus; P_{RE}, Q_{RE}- Active and reactive power at receiving bus; R, X- Resistance and reactance of the transmission line.

3.1 Voltage collapse proximity indicator (VCPI)

The VCPI technique [58, 59] depends upon the maximum power transfer concept of a transmission line. Assuming a load impedance $Z_{LM}\angle\phi$ and a constant voltage source V_{cs} within impedance $Z_{IM}\angle\theta$, the maximum power can be transmitted to the load when the ratio is $Z_{LM}/Z_{IM} = 1.0$. Hence, the ratio is used as the voltage collapse forecaster for that bus after making equivalent the network into a single line with the variables mentioned above. Considering varying load impedance with ϕ remaining constant, with the increment of load demand, Z_{LM} decreases while the current increases. This leads to a voltage drop at the receiving end (RE) as:

$$V_{RE} = Z_{LM} I \tag{2}$$

where

$$\begin{gathered} I = \frac{V_{cs}}{\sqrt{\left[(Z_{IM}\cos\theta + Z_{LM}\cos\phi)^2 + (Z_{IM}\sin\theta + Z_{LM}\sin\phi)^2\right]}} \\ V_{RE} = \frac{Z_{LM}}{Z_{IM}} \frac{V_{cs}}{\sqrt{\left[1 + \left(\frac{Z_{LM}}{Z_{IM}}\right)^2 + 2\left(\frac{Z_{LM}}{Z_{IM}}\right)\cos(\theta - \phi)\right]}} \end{gathered} \tag{3}$$

Active power at RE is:

$$P_{RE} = V_{RE} I \cos\phi \tag{4}$$

$$P_{RE} = \frac{V_{cs}^2/Z_{IM}}{1 + \left(\frac{Z_{LM}}{Z_{IM}}\right)^2 + 2\left(\frac{Z_{LM}}{Z_{IM}}\right)\cos(\theta - \phi)} \frac{Z_{LM}}{Z_{IM}} \cos\phi \tag{5}$$

Accordingly, power loss in the transmission line is:

$$P_{loss} = \frac{V_{cs}^2/Z_{IM}}{1 + \left(\frac{Z_{LM}}{Z_{IM}}\right)^2 + 2\left(\frac{Z_{LM}}{Z_{IM}}\right)\cos(\theta - \phi)} \cos\theta \tag{6}$$

The maximum active power that can be transmitted to the RE can be determined by using the boundary condition $\frac{\partial P_{RE}}{\partial Z_{LM}} = 0$ that leads to $\frac{Z_{LM}}{Z_{IM}} = 1$. Substituting it in (5) yields the maximum transferable power as:

$$P_{\mathrm{RE(\,max)}} = \frac{V_{cs}^2}{Z_{IM}} \frac{\cos\phi}{4\cos^2\left(\frac{\theta - \phi}{2}\right)} \tag{7}$$

Since VCPI depends on the theory of maximum power transmission through a line, VCPI can be defined as:

$$VCPI = \frac{P_{RE}}{P_{RE(\,\max)}} \tag{8}$$

The value of VCPI should be less than 1 for a good voltage stability profile. If it is close to 1.0, it indicates that it is nearing the instability point. Buses approaching an instability point are considered weak buses.

3.2 Line stability index (LSI)

The LSI method [60] is used to determine the stability index for all lines linked between two buses in an electrical network. Using the concept of power flow in the lines of the two-bus system in Fig. 3, power flows at the sending end (SE) and receiving end (RE) can be obtained as:

$$C_{SE} = \frac{|V_{SE}|^2}{Z}\angle\theta - \frac{|V_{SE}||V_{RE}|}{Z}\angle(\theta + \delta_{SE} - \delta_{RE}) \tag{9}$$

$$C_{RE} = \frac{|V_{SE}||V_{RE}|}{Z}\angle(\theta - \delta_{SE} + \delta_{RE}) - \frac{|V_{RE}|^2}{Z}\angle\theta \tag{10}$$

From (9), active and reactive power are separated as:

$$P_{RE} = \frac{V_{SE}V_{RE}}{Z}\cos(\theta - \delta_{SE} + \delta_{RE}) - \frac{V_{RE}^2}{Z}\cos\theta \tag{11}$$

$$Q_{RE} = \frac{V_{SE}V_{RE}}{Z}\sin(\theta - \delta_{SE} + \delta_{RE}) - \frac{V_{RE}^2}{Z}\sin\theta \tag{12}$$

Replacing $\delta_{SE} - \delta_{RE} = \delta$ in (10), V_{RE} can be obtained as:

$$V_{RE} = \frac{V_{SE}\ \sin(\theta - \delta) \pm \sqrt{\{[V_{SE}\ \sin(\theta - \delta)]^2 - 4ZQ_{RE}\ \sin\theta\}}}{2\ \sin\theta} \tag{13}$$

Substituting $Z \sin\theta = x$ yields:

$$V_{RE} = \frac{V_{SE}\ \sin(\theta - \delta) \pm \sqrt{\{[V_{SE}\ \sin(\theta - \delta)]^2 - 4xQ_{RE}\}}}{2\ \sin\theta} \tag{14}$$

For a valid V_{RE} in (14), the following condition must be met and can be used as a stability criterion:

$$[V_{SE}\ \sin(\theta - \delta)]^2 - 4Q_{RE}x \geq 0 \tag{15}$$

Hence, the stability index of the transmission line L_{mn} is:

$$L_{mn} = \frac{4Q_{RE}x}{[V_{SE}\ \sin(\theta - \delta)]^2} \leq 1 \tag{16}$$

If L_{mn} is less than 1, the network is stable while when it is greater than 1, the electrical network is unstable.

3.3 Fast voltage stability index (FVSI)

FVSI [60] is derived by first obtaining a current equation through a line in the two-bus system, as presented in Fig. 3. Taking the sending bus as the reference (i.e., $\delta_{SE} = 0$ and $\delta_{RE} = \delta$), the general current equation can be written as:

$$I = \frac{V_{SE}\angle 0 - V_{RE}\angle\delta}{R + jX} \tag{17}$$

$$V_{RE}^2 - \left(\frac{R}{X}\ \sin\delta +\ \cos\delta\right)V_{SE}V_{RE} + \left(X + \frac{R^2}{X}\right)Q_{SE} = 0 \tag{18}$$

and the RE voltage is:

$$V_{RE} = \frac{\left(\frac{R}{X}\ \sin\delta +\ \cos\delta\right)V_{SE} \pm \sqrt{\left[\left(\frac{R}{X}\ \sin\delta +\ \cos\delta\right)V_{SE}\right]^2 - 4\left(X + \frac{R^2}{X}\right)Q_{RE}}}{2} \tag{19}$$

To find the real value of the receiving end voltage V_{RE}, it must satisfy:

$$\frac{4Z^2Q_{RE}X}{V_{SE}^2(R\ \sin\delta + X\ \cos\delta)^2} \leq 1 \tag{20}$$

Since the phase difference between the SE and RE buses is very small, so $\delta = 0$, $R \sin\delta \approx 0$ and $X \cos\delta \approx X$. Therefore, FVSI can be given as,

$$FVSI = \frac{4Z^2Q_{RE}}{V_{SE}^2X} \tag{21}$$

The FVSI value closing to unity signifies that it nears its vulnerability point, whereas an FVSI greater than 1 implies that the occurrence of a sudden voltage drop in one of the lines will lead to a power network collapse.

3.4 New voltage stability indicator (NVSI)

The NVSI [61] is obtained from the 2-bus network in Fig. 3, ignoring the resistance of the branch. Therefore, NVSI can be given as:

$$NVSI = \frac{2X\sqrt{P_{RE}^2 + Q_{RE}^2}}{2Q_{RE}X - V_{SE}^2} \tag{22}$$

The *NVSI* value must be less than 1 in all power transmission lines to ensure the stability of the electrical grid.

4 Weak bus-constrained PMU placement problem based on BILP methodology

Integer programming (IP) is numerical optimization programming for issues having integer variables, and it is the most common method for unraveling the OPP problem. IP is called integer linear programming (ILP) when the constraints and objective function are linear. In an ILP, when some variables are integers and others non-integers, the ILP is called mixed-ILP (MILP). In the case where the variables are confined within [0, 1], the ILP can be treated as a binary-ILP (BILP) technique. Therefore, constraints play a significant role when using the ILP method to unravel the OPP issue [62]. In this work, the BILP is proposed for solving the OPP problem with and without considering the weakest buses.

The present work has two objectives, i.e., to propose a technique for finding the strategic location of PMU placement and to ensure that the identified weakest buses are fully observable. In order to find the solution for BILP, the *bintprog* in MATLAB, which is built on the LP-based Branch and Bound algorithm (BB, B&B or BBA) is used to solve the OPP problem. The algorithm was first proposed by A. H. Land and A. G. Doig in 1960 for discrete programming, and it is a general algorithm to find optimal solutions for various optimization problems, especially in discrete and combinatorial optimization. A BBA consists of a systematic enumeration of all candidate solutions, where large subsets of ineffective candidates are fathomed (pruning of branches) using upper and lower bounds of the quantity being optimized. The flowchart of BBA is shown in Fig. 4 while the exact algorithm procedure is shown below.

Algorithm A.1: Pseudo-code of the BBA

```
{Initialization}
$LB := -Inf$
$UB := Inf$
Hoard starting node in expecting node list
while expecting node list is no more unfilled do
    {Node assortment}
    Select a node from the expecting node list
    Eliminate it from the expecting node list
    Determine sub-problem
    if impossible then
        Node is measured
    else if optimum then
        if integer explanation then
            If $obj > LB$ then
                {Healthier integer solution obtained}
                $LB := obj$
                Eliminate nodes $j$ from the list with $UB_j < LB$
            end if
        else
            {Variable assortment}
            Determine variable $y_k$ with small value $u$
            Generate node $jnew$ with limit $y_k \leqslant \lfloor u \rfloor$
            $UB_{jnew} := obj$
            Store node $jnew$ in expecting node list
            Generate node $jnew$ with bound $y_k \geqslant \lceil u \rceil$
            $UB\_{jnew} := obj$
            Store node $jnew$ in expecting node list
        end if
    else
        Halt: issue in unravelling sub-problem
    end if
    $UB = \max_j UB_j$
end while
```

The formulation for developing the constraint equation is given for two attainable conditions: (1) ignoring weak bus measurement and (2) considering weak bus measurement. Figures 4, 5, 6 and 7 show the example 8-bus system to demonstrate the OPP problem considering weak buses [47]. The weak bus constrained PMU placement problem can be formulated as:

$$\min \sum_{k=1}^{n} c_k y_k \tag{23}$$

$$s.t. A(Y) \geq b \tag{24}$$

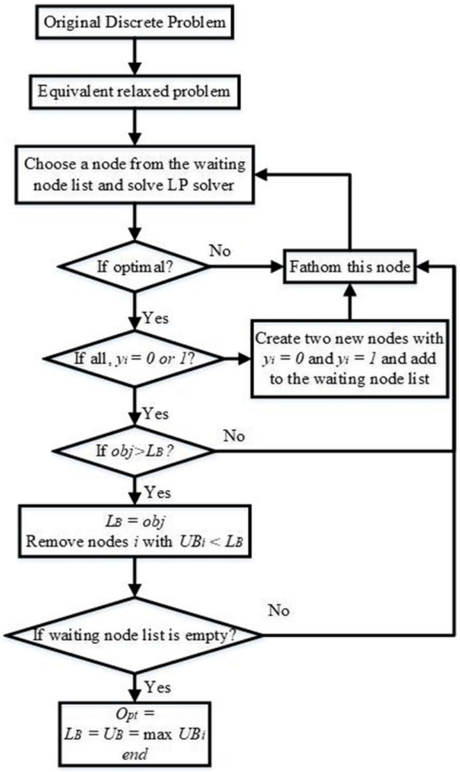

Fig. 4 Flowchart of the BBA method

where n is the total number of buses in the power system for placement of PMUs, c_k is the weight factor of evaluations to the cost of placed PMU at the k^{th} bus, and Y is the binary resolution state vector having an element y_k which determines the attainability of PMU on the k^{th} bus. The binary resolution state vector is given by (25) and $A(Y)$ is the observability constraint whose presence are not zero if the corresponding distant bus voltages are marked with respect to the given sets of evaluations to the above-mentioned procedure. Otherwise its presences are zero.

$$y_k = \begin{cases} 1, & \text{if PMU is required at } k^{th} \text{ bus} \\ 0, & \text{contrarily} \end{cases} \tag{25}$$

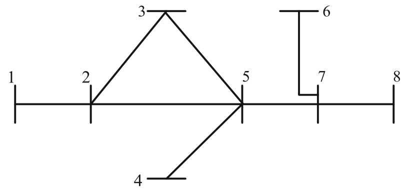

Fig. 5 Illustration diagram of the 8-bus network

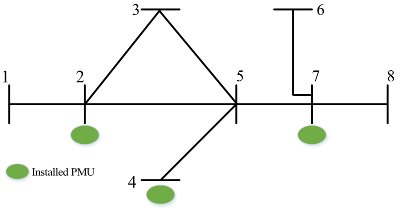

Fig. 6 PMU locations for the 8-bus network

The entries in A are

$$A_{k,i} = \begin{cases} 1, & \text{if bus } k \text{ is linked to bus } i \\ 1, & \text{if } k = i \\ 0, & \text{contrarily} \end{cases} \tag{26}$$

and b is a unit vector given as

$$b^T = [1\ 1\ 1 \ldots 1] \tag{27}$$

4.1 Case 1: without weak bus measurement

The concept of IP is that the installation of PMUs is similar to a problem that reduces the number of PMUs to allow every single bus in the power network to be monitored at least once by the PMUs. So, the objective is to decrease the number of PMUs while still making the power network fully observable. The 8-bus system displayed in Fig. 5 is considered while in this case, weak buses are ignored. To formulate the objective function, the binary bus-to-bus connectivity matrix A is constructed, as interpreted in (26). Defining the design variable to be related with bus i, variable y_k is set to one when an event (PMU installation) occurs at bus i. Otherwise, it is zero. The binary-connectivity matrix $A_{k,\ i}$ for the 8-bus system is given by:

$$A_{(i,k)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{(8\times 8)} \tag{28}$$

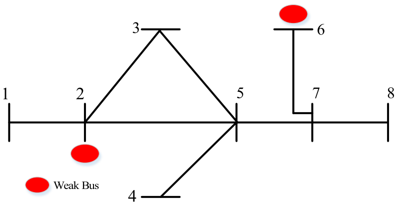

Fig. 7 The 8-bus network with weak buses

The constraint function is formulated as follows:

$$\begin{aligned} &\text{Objective}: \ \min \sum_{i=1}^{8} y_i \\ &\text{Subject to}: \\ &f(Y) = A.Y = \begin{cases} f_1 = y_1 + y_2 & \geq 1 \\ f_2 = y_1 + y_2 + y_3 + y_5 & \geq 1 \\ f_3 = y_2 + y_3 + y_5 & \geq 1 \\ f_4 = y_4 + y_5 & \geq 1 \\ f_5 = y_2 + y_3 + y_4 + y_5 + y_7 & \geq 1 \\ f_6 = y_6 + y_7 & \geq 1 \\ f_7 = y_5 + y_6 + y_7 + y_8 & \geq 1 \\ f_8 = y_7 + y_8 & \geq 1 \end{cases} \\ &y_i \in \{0, 1\} \end{aligned} \tag{29}$$

In (29), '+' means the logical 'OR', and the right-hand side of the inequality means that at least one of the variables present in the sum will be non-zero. The constraints related to f_1 and f_2 are considered as:

$$\left.\begin{aligned} f_1 &= y_1 + y_2 \geq 1 \\ f_2 &= y_1 + y_2 + y_3 + y_5 \geq 1 \end{aligned}\right\} \tag{30}$$

The constraint $f_1 \geq 1$ indicates that a PMU must be located at bus {1} or {2} to observe bus {1}. For $f_2 \geq 1$, a PMU must be placed at any one of the buses {1}, {2}, {3} or {5}, so that bus {2} is observable. Hence, the BILP model is developed as follows:

$$\begin{aligned} &\text{Objective}: \ \min \sum_{i=1}^{8} y_i \\ &\text{Subject to}: \\ &\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\ &y \in \{0, 1\} \end{aligned} \tag{31}$$

The MATLAB function *bintprog* is used to determine the optimal PMU locations, and the solutions are shown in Fig. 6. Under normal operating conditions, the selected PMU set solution by *bintprog* is buses {2, 4, 7}.

4.2 Case 2: with weak bus measurement

Buses {2} and {6} are considered to be weak buses in the 8-bus system, as shown in Fig. 7. These are predicted to be directly observed by placing PMUs at them. In this manner, the constraints for this case are given as:

$$
\begin{aligned}
&\text{Objective}: \ \min \sum_{i=1}^{8} y_i \\
&\text{Subject to}: \\
&f(Y) = \begin{cases}
f_1 = y_1 + y_2 & \geq 1 \\
f_2 = y_1 + y_2 + y_3 + y_5 & \geq 1 \\
f_3 = y_2 + y_3 + y_5 & \geq 1 \\
f_4 = y_4 + y_5 & \geq 1 \\
f_5 = y_2 + y_3 + y_4 + y_5 + y_7 & \geq 1 \\
f_6 = y_6 + y_7 & \geq 1 \\
f_7 = y_5 + y_6 + y_7 + y_8 & \geq 1 \\
f_8 = y_7 + y_8 & \geq 1
\end{cases} \\
&\text{At weak buses}: y_2 = 1, y_6 = 1 \\
&y \in \{0, 1\}
\end{aligned}
\tag{32}
$$

Due to the fact that $y_2 = 1$ and $y_6 = 1$, the inequalities for buses {1}, {2}, {3}, {4}, {5}, {6} and {7} will be immediately fulfilled, and thus these constraints in (32) can be excluded. After reformulating the equations by excluding the satisfied constraints, the BILP problem is given by:

$$
\begin{aligned}
&\text{Objective}: \ \min \sum_{i=1}^{8} y_i \\
&\text{Subject to}: \\
&\left. \begin{aligned} f_4 = y_4 + y_8 \geq 1 \\ f_8 = y_7 + y_8 \geq 1 \end{aligned} \right\} \\
&\text{At weak buses}: y_2 = 1, y_6 = 1. \\
&y \in \{0, 1\}
\end{aligned}
\tag{33}
$$

The optimal set solution is buses {2, 4, 6, 7} as shown in Fig. 8.

The *bintprog* implements a BBA to solve the discrete optimization problem considering weak buses [63, 64] and the flowchart is displayed in Fig. 9. The exact '*bintprog*' procedure is shown below.

```
Algorithm A.2: Procedure of bintprog
$c_k$ : Implementation cost of PMUs
$n$ : Number of nodes (buses)
$l$ : Number of branches
Define variables: $Y = [y_1, \ y_2, \ \ldots, \ y_n]^T \ y_i \in \{0, \ 1\}^n$
Define bus-to-bus connection matrix
Define the weakest bus
$f(Y) = c_k y$ : Objective function considering the cost
/* Determining the problem of discrete problem*/
$Y$ = bintprog $(f, -A, -b)$; $b$: unit vector, $y_i \in \{0,1\}^n$
$Y$ : Optimal solution for PMUs
Solution $Y$
```

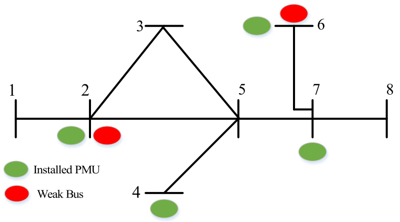

Fig. 8 Weak bus constrained PMU placement problem solutions

5 Case studies and detailed results

The proposed formulation for determining the strategic locations of PMUs has been conducted on the standard IEEE 14-, 118-, 300-bus, and NE 39-bus test systems. MATLAB based *bintprog* software package is used to perform the BILP program and implements BBA for identification of the binary issues.

The following steps illustrate the use of the BILP method: (i) Read the standard information; (ii) Obtain the discrete vertex-to-vertex matrix $A_{i,\ k}$ and the PMU price coefficient vector *c*; (iii) Select the weakest buses; (iv) Configure the right-hand side unit vector of (A.7); (v) Determine the BILP problem using *bintprog*. The flowchart is shown in Fig. 10 for the identification of the OPP problem considering weak buses.

The technical configurations of the computer used for the simulations are: Intel core I3-5005U (2.0 GHz), L3 Cache 3 MB, and System Memory 8GB DDR3. Table 1 displays the number and locations of ZIBs and RBs for the standard test systems. The optimal PMU locations and computational times with and without considering ZIBs during usual operating conditions are given in Table 2. In the results shown in Table 2, the weak buses are not considered for the OPP problem.

Weak lines are heavily loaded lines in a power network. Due to load variation, the line capacity can be exceeded which may cause severe damage in

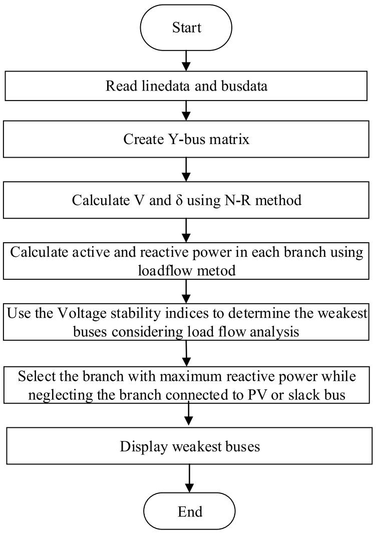

Fig. 9 Flow chart for obtaining the weakest bus in a connected power network

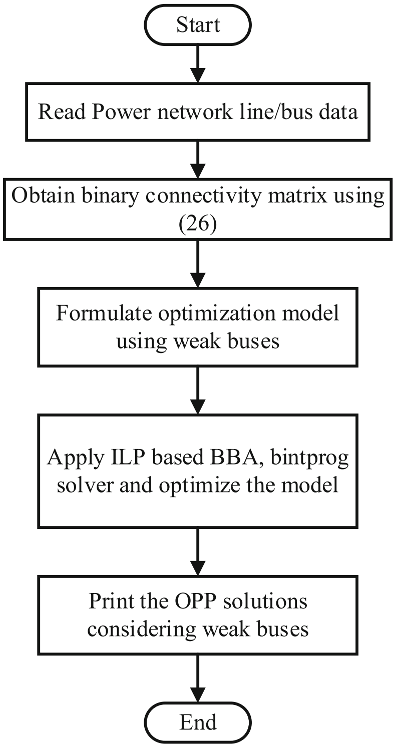

Fig. 10 BILP flowchart for a *bintprog* solver for the OPP problem

transmission lines. The weak bus measurement methods, as explained in Section 3, are used to identify the weak buses. The different voltage stability indices are also clarified in Section 3. The main aim of the voltage stability indices is to compute how close a specific point is to the steady-state voltage stability margin. These voltage stability indices can be used on-line or off-line to help the operators in real-time operation of connected power networks. These indices can determine how close to voltage instability a network can be operated.

Four voltage stability indices are considered here, although there are many indices to examine the weakest buses. The weakest buses are identified by the following steps: a) Carry out load flow analysis (using MATPOWER 5.1) and find voltage (V), active (P) and reactive power (Q) of the buses; b) Utilize V, P, and Q to find the weak buses based on weak bus measurement methods such as VCPI, LSI, FVSI, and NVSI.

The weakest buses obtained with voltage stability indices are shown in Table 3 for different test systems. Here, buses connected either with generators or shunt capacitors are not considered as weak buses. In the case of the IEEE 14-bus system, the weak buses are {5, 11, 13} using VCPI, {7, 11, 13} using both LSI and FVSI, and {10, 11, 13} using NVSI. Weak buses for other test systems shown in Table 3 can be obtained in similar ways. It is to be noted that different weakest buses are obtained.

Table 4 displays the optimal locations of PMUs considering weak bus measurements. For the 14-bus system, the optimal locations of PMUs considering weak buses are {3, 5, 7, 10, 13} for all the VCPI, LSI, FVSI, and NVSI methods. Similarly, for the 39-bus system, the optimal number of PMU locations is 14 for the different weak bus measurement methods, i.e. {2, 4, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29}. In the case of the 118-bus system, 32 PMUs are considered using different weak bus methods and the optimal PMU locations are {3, 6, 9, 11, 12, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114}. For the 300-bus system, 88 PMUs are considered for placement. It is to be noted that the PMU locations for all the voltage stability indices are the same in number but with different positions.

The optimal numbers of PMUs obtained using the proposed approach are compared with the existing standard results found in the literature in Table 5.

Reference [65] proposes 5 optimal locations of PMU placement in the IEEE 14-bus test system using a simulated annealing method considering weak buses.

Table 1 Number and locations of ZIBs and RBs for standard test systems

Test system	Size (bus, lines)	No. of ZIBs	Loc. of ZIBs	No. of RBs	Loc. of RBs
14- bus	14,20	1	7	1	8
39- bus	39,46	11	1, 2, 5, 6, 9, 11, 13, 14, 17, 19, 22	9	30, 31, 32, 33, 34, 35, 36, 37, 38
118- bus	118,186	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	7	10, 73, 87, 111, 112, 116, 117
300- bus	300, 411	65	4, 7, 12, 16, 18, 23, 28, 29, 30, 33, 36, 39, 40, 52, 54, 56, 57, 62, 65, 70, 71, 72, 73, 82, 94, 95, 96, 107, 108, 109, 110, 111, 112, 113, 123, 129, 130, 137, 139, 143, 144, 145, 147, 148, 153, 172, 173, 174, 189, 191, 198, 205, 216, 219, 223, 245, 246, 266, 270, 271, 272, 273, 276, 291	69	69, 150, 164, 192, 201, 206, 209, 212, 215, 218, 220, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296, 297, 298, 299, 300

Table 2 Minimum number and optimal locations of PMUs, and computational time under normal operating conditions ignoring weak buses

Test system	Status of ZIBs	# PMU	Optimal PMU locations	Comp. time/s
14- bus	Ig. ZIBs	4	2 6 7 9	0.042
	Cons. ZIBs	3	2 6 9	0.056
39- bus	Ig. ZIBs	13	2 6 9 10 13 14 17 19 20 22 23 25 29	0.661
	Cons. ZIBs	8	3 8 10 16 20 23 25 29	0.154
118- bus	Ig. ZIBs	32	3 7 9 11 12 17 21 25 28 34 37 41 45 49 53 56 62 63 68 70 71 76 79 85 86 89 92 96,100,105,110,114	0.140
	Cons. ZIBs	27	2 12 15 17 21 23 28 34 37 40 45 49 52 62 63 68 71 75 77 80 85 90 94,101,105,110,114	0.403
300- bus	Ig. ZIBs	87	1, 2, 3, 11, 15, 19, 23, 25, 27, 33, 37, 43, 48, 49, 53, 54, 62, 64, 68, 69, 71, 73, 76, 83, 86, 88, 92, 93, 98, 99, 101, 109, 111, 112, 113, 116, 119, 124, 132, 135, 138, 139, 143, 145, 152, 157, 163, 167, 183, 187, 188, 189, 190, 193, 196, 202, 204, 209, 210, 212, 213, 216, 218, 222, 226, 228, 230, 233, 236, 237, 238, 242, 251, 252, 253, 256, 262, 264, 268, 269, 270, 272, 275, 276, 277, 299, 300	1.249
	Cons. ZIBs	77	6, 17, 26, 53, 56, 69, 73, 87, 92, 103, 110, 117, 123, 126, 128, 129, 135, 140, 145, 147, 148, 149, 150, 153, 158, 159, 162, 164, 186, 187, 188, 189, 209, 212, 215, 216, 218, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296, 297, 298, 299, 300	1.943

However, the proposed approach may not provide a global optimal solution for larger electrical systems. In the present work, 5 locations are also determined for PMU installation in the 14-bus system using BILP considering weak buses, while PMU locations are obtained for the 39-, 118-, and 300-bus systems. The computational times shown also demonstrate the efficacy and robustness of the proposed approach. In conclusion, the proposed approach is very efficient in determining the optimal PMU locations considering weak buses for large power networks.

It can be observed that the optimal numbers of PMUs considering weak bus measurements required to make the connected power networks (IEEE 14-, 118-, 300-, and NE 39-bus system) fully observable are similar to the results published in the open literature.

Table 3 Identification of weak buses

Test system	Indices	# weak buses	Loc. of weak bus	Comp. time/s
14- bus	VCPI	3	5, 11, 13	0.015
	LSI	3	7, 11 13	0.013
	FVSI	3	7, 11, 13	0.199
	NVSI	3	10, 11, 13	0.028
39- bus	VCPI	4	6, 8, 11, 13	0.123
	LSI	4	16, 19, 20, 22	0.125
	FVSI	4	9, 11, 13, 22	0.122
	NVSI	4	18, 23, 24, 27	0.043
118- bus	VCPI	6	17, 37, 64, 79, 81, 115	0.100
	LSI	6	23, 29, 64, 86, 96, 108	0.097
	FVSI	6	5, 23, 37, 94, 96, 97	0.154
	NVSI	6	29, 58, 79, 96, 109, 115	0.147
300- bus	VCPI	10	5, 7, 65, 97, 160, 167, 216, 230, 266, 270	0.186
	LSI	10	73, 78, 91, 189, 198, 229, 231, 245, 246, 283	0.212
	FVSI	10	15, 79, 90, 100, 102, 153, 158, 196, 245, 246	0.266
	NVSI	10	67, 93, 99, 179, 216, 230, 266, 269, 270, 291	0.235

Table 4 Optimal PMU locations considering weak buses

Test system	Indices	# PMUs	Weak bus constrained PMU location using BILP	Comp. time/s
14- bus	VCPI	5	3, 5, 7, 10, 13	0.048
	LSI	5	3, 5, 7, 10, 13	0.671
	FVSI	5	3, 5, 7, 10, 13	0.571
	NVSI	5	3, 5, 7, 10, 13	0.768
39- bus	VCPI	14	2, 4, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29	0.373
	LSI	14	2, 4, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29	0.879
	FVSI	14	2, 4, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29	0.810
	NVSI	14	2, 4, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29	0.837
118- bus	VCPI	32	3, 6, 9, 11, 12, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	0.868
	LSI	32	3, 6, 9, 11, 12, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	0.898
	FVSI	32	3, 6, 9, 11, 12, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	0.894
	NVSI	32	3, 6, 9, 11, 12, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	0.813
300- bus	VCPI	88	1, 2, 3, 10, 11, 15, 19, 23, 25, 27, 33, 37, 43, 48, 49, 53, 54, 62, 64, 68, 69, 71, 73, 76, 83, 86, 88, 92, 93, 98, 99, 101, 109, 111, 112, 113, 116, 119, 124, 132, 135, 138, 139, 143, 145, 152, 157, 163, 167, 183, 187, 188, 189, 190, 193, 196, 202, 204, 209, 210, 212, 213, 216, 218, 222, 226, 228, 230, 233, 236, 237, 238, 242, 251, 252, 253, 256, 262, 264, 268, 269, 270, 272, 275, 276, 277, 299, 300	1.351
	LSI	88	1, 2, 3, 10, 11, 15, 19, 23, 25, 27, 33, 37, 43, 48, 49, 53, 54, 62, 64, 68, 69, 71, 73, 76, 83, 86, 88, 92, 93, 98, 99, 101, 109, 111, 112, 113, 116, 119, 124, 132, 135, 138, 139, 143, 145, 152, 157, 163, 167, 183, 187, 188, 189, 190, 193, 196, 202, 204, 209, 210, 212, 213, 216, 218, 222, 226, 228, 230, 233, 236, 237, 238, 242, 251, 252, 253, 256, 262, 264, 268, 269, 270, 272, 275, 276, 277, 299, 300	1.149
	FVSI	88	1, 2, 3, 10, 11, 12, 15, 17, 22, 23, 25, 26, 27, 33, 37, 38, 43, 48, 49, 53, 54, 55, 58, 59, 60, 62, 64, 65, 68, 71, 73, 79, 83, 85, 86, 88, 92, 93, 98, 99, 101, 109, 111, 112, 113, 116, 118, 119, 128, 132, 135, 138, 139, 143, 145, 152, 157, 163, 167, 173, 183, 187, 188, 189, 190, 193, 196, 202, 204, 208, 210, 211, 213, 216, 217, 219, 222, 226, 228, 267, 268, 269, 270, 272, 273, 274, 276, 294	1.305
	NVSI	88	1, 2, 3, 10, 11, 12, 15, 17, 22, 23, 25, 26, 27, 33, 37, 38, 43, 48, 49, 53, 54, 55, 58, 59, 60, 62, 64, 65, 68, 71, 73, 79, 83, 85, 86, 88, 92, 93, 98, 99, 101, 109, 111, 112, 113, 116, 118, 119, 128, 132, 135, 138, 139, 143, 145, 152, 157, 163, 167, 173, 183, 187, 188, 189, 190, 193, 196, 202, 204, 208, 210, 211, 213, 216, 217, 219, 222, 226, 228, 267, 268, 269, 270, 272, 273, 274, 276, 294	1.484

6 Conclusion

In this work, BILP programming has been used to determine the minimum number and optimal PMU locations while considering the weak buses for full observability of the power network. The weak bus measurement methods such as VCPI, LSI, FVSI, and NVSI are used to identify the weak buses in the power network. The number of PMU placements required to make the power network completely observable is provided by the topology analysis method that depends on the binary bus-to-bus connectivity matrix. BILP is then used to ensure that the weakest buses are directly observable. The proposed methodology has been tested on the standard IEEE 14-, 118-, 300-bus and NE 39-bus systems and the results are compared with the existing methods. They have shown that the present methodology can accomplish the weak bus-constrained PMU placements for full power network observability.

Table 5 Comparative analysis of obtained results with the existing method

Test system	Proposed approach				SA [65]
	VCPI	LSI	FVSI	NVSI	BSA
14- bus	5	5	5	5	5
39- bus	14	14	14	14	–
118- bus	32	32	32	32	–
300- bus	88	88	88	88	–

(–-) entry means not reported
BSA Branch Sensitivity Analysis Method

Abbreviations

PMU: Phasor measurement unit; SCADA: Supervisory control and data acquisition; KCL: Kirchhoff's Current Law; OPP: Optimal PMU placement;

IP: Integer programming; ILP: Integer linear programming; MILP: Mixed integer linear programming; BILP: Binary integer linear programming; BBA: Branch and Bound algorithm; DOU1: Depth-of-Unobservability one; DOU2: Depth-of-Unobservability two; SORI: System observability redundancy index; SQP: Sequential quadratic programming; BSDP: Binary semi-definite programming; VCPI: Voltage collapse proximity indicator; LSI: Line stability index; FVSI: Fast voltage stability index; NVSI: New voltage stability index; MATPOWER: Open-Source Matlab-Language M-Files for Solving Steady-State Power System Simulation and Optimization Problems; SE: Sending end; RE: Receiving end; ZIB: Zero injection bus; RB: Radial bus; bintprog: Binary integer linear programming MATLAB toolbox

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Authors' contributions
RB analyzed and interpreted the data regarding the optimal PMU placement (OPP) problem and the installation of PMU. SR performed the load flow analysis to determine the weak lines for IEEE systems. RB also performed the examination of the OPP problem and solve the problem using the deterministic method, and was a major contributor in writing the manuscript. The overall manuscript is checked by BB. All authors read and approved the final manuscript.

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Availability of data and materials
The authors declare that the data supporting the findings of this study are available within the article. For example, IEEE systems and other MATLAB toolboxes are given in the reference section.

Competing interests
The authors declare that they have no competing interests.

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