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Opposition-based differential evolution for hydrothermal power system

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Abstract

This paper presents opposition-based differential evolution to determine the optimal hourly schedule of power generation in a hydrothermal system. Differential evolution (DE) is a population-based stochastic parallel search evolutionary algorithm. Opposition-based differential evolution has been used here to improve the effectiveness and quality of the solution. The proposed opposition-based differential evolution (ODE) employs opposition-based learning (OBL) for population initialization and also for generation jumping. The effectiveness of the proposed method has been verified on two test problems, two fixed head hydrothermal test systems and three hydrothermal multi-reservoir cascaded hydroelectric test systems having prohibited operating zones and thermal units with valve point loading. The results of the proposed approach are compared with those obtained by other evolutionary methods. It is found that the proposed opposition-based differential evolution based approach is able to provide better solution.

Keywords: Differential evolution, Opposition-based differential evolution, Hydrothermal system, Fixed head, Variable head

Introduction

Optimal scheduling of power plant generation is of great importance to electric utility systems. Because of insignificant marginal cost of hydroelectric power, the problem of minimizing the operational cost of hydrothermal system essentially reduces to that of minimizing the fuel cost of thermal plants under the various constraints on the hydraulic, thermal and power system network.

The hydrothermal scheduling problem has been the subject of investigation for several decades. Several classical methods such as Newton's method [1], mixed integer programming [2, 3], dynamic programming (DP) [4], etc. have been widely used to solve hydrothermal scheduling problem. Among these methods, DP appears to be the most popular. However, major disadvantages of DP method are computational and dimensional requirements which grow drastically with increasing system size and planning horizon.

Recently, stochastic search algorithms such as simulated annealing (SA) [5], evolutionary programming (EP) [6], genetic algorithm (GA) [7, 8], evolutionary programming technique [9], differential evolution (DE) [10–12], particle swarm optimization [13], artificial immune system [14], clonal selection algorithm [15] and teaching learning based optimization [16] have been successfully used to solve hydrothermal scheduling problem.

Since the mid 1990s, many techniques originated from Darwin's natural evolution theory have emerged. These techniques are usually termed by "evolutionary computation methods" including evolutionary algorithms (EAs), swarm intelligence and artificial immune system. Differential evolution (DE) [17–20], a relatively new member in the family of evolutionary algorithms, first proposed over 1995–1997 by Storn and Price at Berkeley is a novel approach to numerical optimization. It is a population-based stochastic parallel search evolutionary algorithm which is very simple yet powerful. The main advantages of DE are its capability of solving optimization problems which require minimization process with nonlinear, non-differentiable and multi-modal objective functions.

The basic concept of opposition-based learning (OBL) [21–23] was originally introduced by Tizhoosh. The

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main idea behind OBL is for finding a better candidate solution and the simultaneous consideration of an estimate and its corresponding opposite estimate (i.e., guess and opposite guess) which is closer to the global optimum. OBL was first utilized to improve learning and back propagation in neural networks by Ventresca and Tizhoosh [24], and since then, it has been applied to many EAs, such as differential evolution [25], particle swarm optimization [26] and ant colony optimization [27].

Opposition-based harmony search algorithm [28] has been applied to solve combined economic and emission dispatch problems. In [29] oppositional real coded chemical reaction optimization has been used for solving economic dispatch problems. Opposition-based gravitational search algorithm [30] has been applied for solving reactive power dispatch problem.

This paper proposes opposition-based differential evolution (ODE) for optimal scheduling of generation in a hydrothermal system. This paper considers fixed head as well as variable head hydrothermal system. In case of fixed head hydro plants, water discharge rate curves are modeled as a quadratic function of the hydropower generation and thermal units with non-smooth fuel cost function. Here, scheduling period is divided into a number of subintervals each having a constant load demand. In case of variable head hydrothermal system, multi-reservoir cascaded hydro plants having prohibited operating zones and thermal units with valve point loading are used. The proposed method is validated by applying it to two test problems, two fixed head hydrothermal test systems and three hydrothermal multi-reservoir cascaded hydroelectric test systems having prohibited operating zones and thermal units with valve point loading. The test results are compared with those obtained by other evolutionary methods reported in the literature. From numerical results, it is found that the proposed ODE based approach provides better solution.

Problem formulation

Fixed head hydrothermal system

Fixed head hydrothermal scheduling problem with N_h hydro units and N_s thermal units over M time subintervals is described as follows:

Objective function

The fuel cost function of each thermal generator, considering valve-point effect, is expressed as a sum of quadratic and sinusoidal function. The superimposed sine components represent rippling effect produced by steam admission valve opening. The problem minimizes following total fuel cost

$$f_{FH} = \sum_{m=1}^{M}\sum_{i=1}^{N_s} t_m \left[a_{si} + b_{si}P_{sim} + c_{si}P_{sim}^2 + d_{si} \times \sin\left\{e_{si} \times \left(P_{si}^{\min} - P_{sim}\right)\right\}\right] \tag{1}$$

Constraints

(i) Power balance constraints:

$$\sum_{i=1}^{N_s} P_{sim} + \sum_{j=1}^{N_h} P_{hjm} - P_{Dm} - P_{Lm} = 0 \quad m\in M \tag{2}$$

and

$$P_{Lm} = \sum_{l=1}^{N_h+N_s}\sum_{r=1}^{N_h+N_s} P_{lm}B_{lr}P_{rm} \quad m\in M \tag{3}$$

(ii) Water availability constraints:

$$\sum_{m=1}^{M}\left[t_m\left(a_{0hj} + a_{1hj}P_{hjm} + a_{2hj}P_{hjm}^2\right)\right] - W_{hj} = 0 \quad j\in N_h \tag{4}$$

(iii) Generation limits:

$$P_{hj}^{\min} \le P_{hjm} \le P_{hj}^{\max} \qquad j\in N_h,\ \ m\in M \tag{5}$$

and

$$P_{si}^{\min} \le P_{sim} \le P_{si}^{\max} \qquad i\in N_s,\ \ m\in M \tag{6}$$

Determination of generation level of slack generator

Thermal generators and hydro generators deliver their power output subject to the power balance constraint (2), water availability constraint (4) and respective capacity constraints (5) and (6). Assuming the power loading of N_P and first (N_s - 1) generators are known, the power level of the N_s th generator (i.e. the slack generator) is given by

$$P_{N_s m} = P_{Dm} + P_{Lm} - \sum_{l=1}^{N_h+N_s-1} P_{lm} \qquad m\in M \tag{7}$$

The transmission loss P_{Lm} is a function of all the generators including the slack generator and it is given by

$$\begin{aligned} P_{Lm} = &\sum_{l=1}^{N_h+N_s-1}\sum_{r=1}^{N_h+N_s-1} P_{lm}B_{lr}P_{rm} \\ &+ 2P_{N_s m}\left(\sum_{l=1}^{N_h+N_s-1} B_{N_s l}P_{lm}\right) \\ &+ B_{N_s N_s}P_{N_s m}^2 \qquad m\in M \end{aligned} \tag{8}$$

Expanding and rearranging, equation (7) becomes

$$
\begin{aligned}
& B_{N_sN_s}P_{N_sm}^2 + \left(2\sum_{l=1}^{N_h+N_s-1} B_{N_sl}P_{lm}-1\right)P_{N_sm} + P_{Dm} \\
& + \sum_{l=1}^{N_h+N_s-1}\sum_{r=1}^{N_h+N_s-1} P_{lm}B_{lr}P_{rm} - \sum_{l=1}^{N_h+N_s-1} P_{lm} \\
& = 0 \qquad m\in M
\end{aligned} \tag{9}
$$

The loading of the slack generator (i.e. N_s th) can then be found by solving equation (9) using standard algebraic method.

Variable head hydrothermal system

The variable head hydrothermal scheduling problem is aimed to minimize the fuel cost of thermal plants, while making use of the availability of hydro power as much as possible. The objective function and associated constraints of the hydrothermal scheduling problem are formulated as follows.

Objective function

$$
\begin{aligned}
\text{Minimize } f_{VH} = \sum_{t=1}^{T}\sum_{i=1}^{N_s}\Big[& a_{si} + b_{si}P_{sit} + c_{si}P_{sit}^2 \\
& + \left|d_{si} \times \sin\left\{e_{si} \times \left(P_{si}^{\min} - P_{sit}\right)\right\}\right|\Big]
\end{aligned} \tag{10}
$$

Constraints

(i) Power balance constraints:

The total active power generation must balance the predicted power demand and transmission loss, at each time interval over the scheduling horizon

$$\sum_{i=1}^{N_s} P_{sit} + \sum_{j=1}^{N_h} P_{hjt} - P_{Dt} - P_{Lt} = 0 \qquad t\in T \tag{11}$$

The hydroelectric generation is a function of water discharge rate and reservoir water head, which in turn, is a function of storage.

$$
\begin{aligned}
P_{hjt} = {} & C_{1j}V_{hjt}^2 + C_{2j}Q_{hjt}^2 + C_{3j}V_{hjt}Q_{hjt} \\
& + C_4V_{hjt} + C_{5j}Q_{hjt} \\
& + C_{6j} \quad j\in N_h \qquad t\in T
\end{aligned} \tag{12}
$$

The transmission loss P_{Lt} is given by

$$P_{Lt} = \sum_{i=1}^{N_s+N_h}\sum_{j=1}^{N_s+N_h} P_{it}B_{ij}P_{jt} + \sum_{i=1}^{N_s+N_h} B_{0i}P_{it} + B_{00} \tag{13}$$

(ii) Generation limits:

$$P_{hj}^{\min} \le P_{hjt} \le P_{hj}^{\max}, \qquad j\in N_h, \quad t\in T \tag{14}$$

and

$$P_{si}^{\min} \le P_{sit} \le P_{si}^{\max}, \qquad i\in N_s, \quad t\in T \tag{15}$$

(iii) Hydraulic network constraints

The hydraulic operational constraints comprise the water balance equations for each hydro unit as well as the bounds on reservoir storage and release targets. These bounds are determined by the physical reservoir and plant limitations as well as the multipurpose requirements of the hydro system. These constraints include:

(a) Physical limitations on reservoir storage volumes and discharge rates,

$$V_{hj}^{\min} \le V_{hjt} \le V_{hj}^{\max}, \qquad j\in N_h, \qquad t\in T \tag{16}$$

$$Q_{hj}^{\min} \le Q_{hjt} \le Q_{hj}^{\max}, \qquad j\in N_h, \qquad t\in T \tag{17}$$

b) The continuity equation for the hydro reservoir network

$$
\begin{aligned}
V_{hj(t+1)} = {} & V_{hjt} + I_{hjt} - Q_{hjt} - S_{hjt} \\
& + \sum_{l=1}^{R_{uj}}\left(Q_{hl\left(t-\tau_{lj}\right)} + S_{hl\left(t-\tau_{lj}\right)}\right), \quad j\in N_h, \quad t\in T
\end{aligned} \tag{18}
$$

(iv) Prohibited operating regions of water discharge-rates

$$
Q_{hj} \in \begin{cases}
{Q_{hj}}^{\min} \le Q_{hj} \le {Q_{hj,1}}^{L} \\
{Q_{hj,k-1}}^{U} \le Q_{hj} \le {Q_{hj,k}}^{L}, k = 2, \ldots n_j \\
{Q_{hj,n_j}}^{U} \le Q_{hj} \le {Q_{hj}}^{\max}
\end{cases} \tag{19}
$$

Description of opposition-based differential evolution

A brief description of differential evolution

Differential Evolution (DE) is a type of evolutionary algorithm originally proposed by Price and Storn [19] for optimization problems over a continuous domain. DE is exceptionally simple, significantly faster and robust. The basic idea of DE is to adapt the search during the evolutionary process. At the start of the evolution, the perturbations are large since parent populations are far away from each other. As the evolutionary process matures, the population converges to a small region and the perturbations adaptively become small. As a result, the evolutionary algorithm performs a global exploratory search during the early stages of the evolutionary process and local exploitation during the mature stage of the search. In DE the fittest of an offspring competes one-to-one with that of corresponding parent which is different from other evolutionary algorithms. This one-to-one competition gives rise to faster convergence rate. Price and Storn gave the working principle of DE with simple strategy in [19]. Later on, they suggested ten different strategies of DE [18]. Strategy-7 (DE/rad/1/bin) is the most successful and widely used strategy. The key parameters of control in DE are population size (N_P), scaling factor (F) and crossover rate (C_R). The optimization process in DE is carried out with three basic operations: mutation, crossover and selection. The DE algorithm is described as follows:

Initialization

The initial population of N_P vectors is randomly selected based on uniform probability distribution for all variables to cover the entire search uniformly. Each individual X_i is a vector that contains as many parameters as the problem decision variables D. Random values are assigned to each decision parameter in every vector according to:

$$X_{ij}^{0} \sim U\left(X_j^{\min}, X_j^{\max}\right) \tag{20}$$

where i =1,…., N_P and j =1,…., D; $X_j^{\min}$ and $X_j^{\max}$ are the lower and upper bounds of the j th decision variable; $U\left(X_j^{\min}, X_j^{\max}\right)$ denotes a uniform random variable ranging over $\left[X_j^{\min}, X_j^{\max}\right]$. X_{ij}^0 is the initial j th variable of i th population. All the vectors should satisfy the constraints. Evaluate the value of the cost function $f\left(X_i^0\right)$ of each vector.

Mutation

DE generates new parameter vectors by adding the weighted difference vector between two population members to a third member. For each target vector X_i^k at k th iteration the noisy vector $X_{i/k}$ is obtained by

$$X_{i/k} = X_a^k + F\left(X_b^k - X_c^k\right), \qquad i \in N_P \tag{21}$$

where X_a^{ka}, X_b^k and X_c^k are selected randomly from N_P vectors at k th iteration and $a \neq b \neq c \neq i$. The scaling factor (F), in the range $0 < F \leq 1.2$, controls the amount of perturbation added to the parent vector. The noisy vectors should satisfy the constraint.

Crossover

Perform crossover for each target vector X_i^k with its noisy vector $X_{i/k}$ and create a trial vector $X_{i//k}$ such that

$$X_{i//k} = \begin{cases} X_{i/k}, & \text{if} \quad \rho \leq C_R \\ X_i^k, & \text{otherwise} \end{cases}, i \in N_P \tag{22}$$

where ρ is an uniformly distributed random number within [0, 1]. The crossover constant (C_R), in the range $0 \leq C_R \leq 1$, controls the diversity of the population and aids the algorithm to escape from local optima.

Selection

Perform selection for each target vector, X_i^k by comparing its cost with that of the trial vector, $X_{i//k}$. The vector that has lesser cost of the two would survive for the next iteration.

$$X_i^{k+1} = \begin{cases} X_{i//k}, & \text{if} \quad f\left(X_{i//k}\right) \leq f\left(X_i^k\right) \quad i \in N_P \\ X_i^k, & \text{otherwise} \end{cases} \tag{23}$$

The process is repeated until the maximum number of iterations or no improvement is seen in the best individual after many iterations.

Figure 1 shows the flowchart of differential evolution.

Opposition-based learning

Opposition-based learning (OBL) was developed by Tizhoosh to improve candidate solution by considering

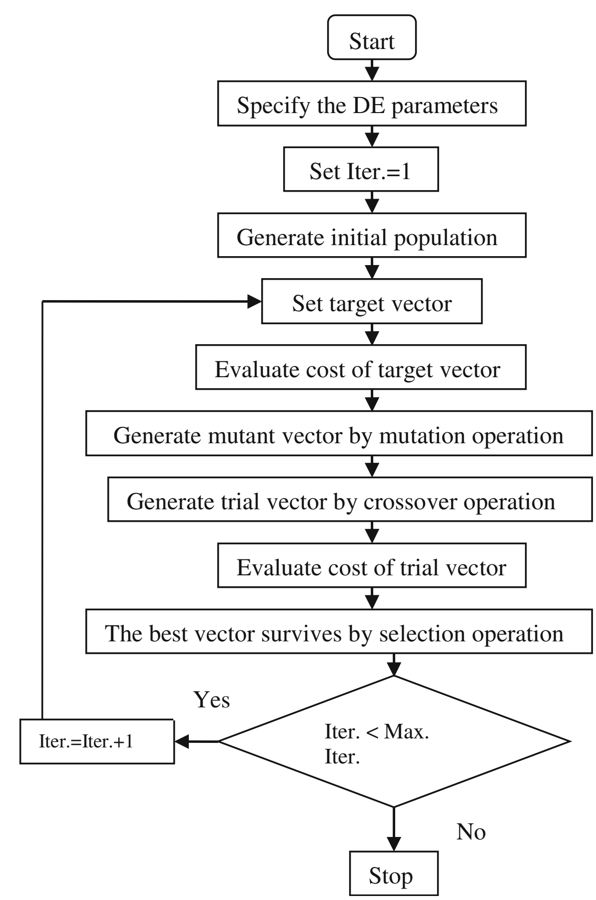

Fig. 1 Flowchart of differential evolution

current population as well as its opposite population at the same time.

Evolutionary optimization methods start with some initial population and try to improve them toward some optimal solution. The process of searching terminates when some predefined criteria are satisfied. The process is started with random guesses in the absence of a priori information about the solution. The process can be improved by starting with a closer i.e. fitter solution by simultaneously checking the opposite solution. By doing this, the fitter one (guess or opposite guess) may be chosen as an initial solution. According to the theory of probability, 50% of the time, a guess is further from the solution than its opposite guess. Therefore, process starts with the closer of the two guesses. The same approach can be applied not only to the initial solution but also continuously to each solution in the current population.

Definition of opposite number

If x be a real number between $[lb, ub]$, its opposite number is defined as

$$\overline{x} = lb + lu - x \tag{24}$$

Similarly, this definition can be extended to higher dimensions [21] as stated in the next sub-section.

Definition of opposite point

Let $X = (x_1, x_2, ..., x_n)$ be a point in n - dimensional space where $x_i \in [lb_i, ub_i]$ and $i \in 1, 2, \ldots, n$. The opposite point $\overline{X} = (\overline{x_1}, \overline{x_2}, \ldots.., \overline{x_n})$ is completely defined by its components as in (25).

$$\overline{x_i} = lb_i + ub_i - x_i \tag{25}$$

By employing the definition of opposite point, the opposition-based optimization is defined in the following sub-section.

Opposition-based optimization

Let $X = (x_1, x_2, ..., x_n)$ be a point in n - dimensional space i.e. a candidate solution. Assume $f = (\bullet)$ is a fitness function which is used to measure the candidate's fitness. According to the definition of the opposite point, $\overline{X} = (\overline{x_1}, \overline{x_2}, \ldots.., \overline{x_n})$ is the opposite of $X = (x_1, x_2, ..., x_n)$. Now, if $f(\overline{X}) < f(X)$ (for a minimization problem), then point X can be replaced with $\overline{X}$; otherwise, the process is continued with X. Hence, the point and its opposite point are evaluated simultaneously in order to continue with the fitter one.

Opposition-based differential evolution

In the present work, the concept of the opposition-based learning [21] is incorporated in differential evolution. The original DE is chosen as a parent algorithm and the opposition-based ideas are embedded in DE.

Figure 2 shows the flowchart of ODE algorithm.

Simulation results

Two test problems, two fixed head hydrothermal systems and three hydrothermal multi-reservoir cascaded hydroelectric test systems having prohibited operating zones and thermal units with valve point loading are investigated. The computational results have been used to compare the performance of the proposed ODE method with that of other evolutionary methods. The proposed ODE algorithm and DE algorithm used in this paper are implemented by using MATLAB 7.0 on a PC (Pentium-IV, 80 GB, 3.0 GHz).

Simple examples

Example 1: Consider the maximization problem [31]

$$\max_{x_1, x_2} f(x_1, x_2) = 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2) \tag{26}$$

where $-3.0 \le x_1 \le 12.1$ and $4.1 \le x_2 \le 5.8$

This function is multimodal. The problem is solved by using ODE.

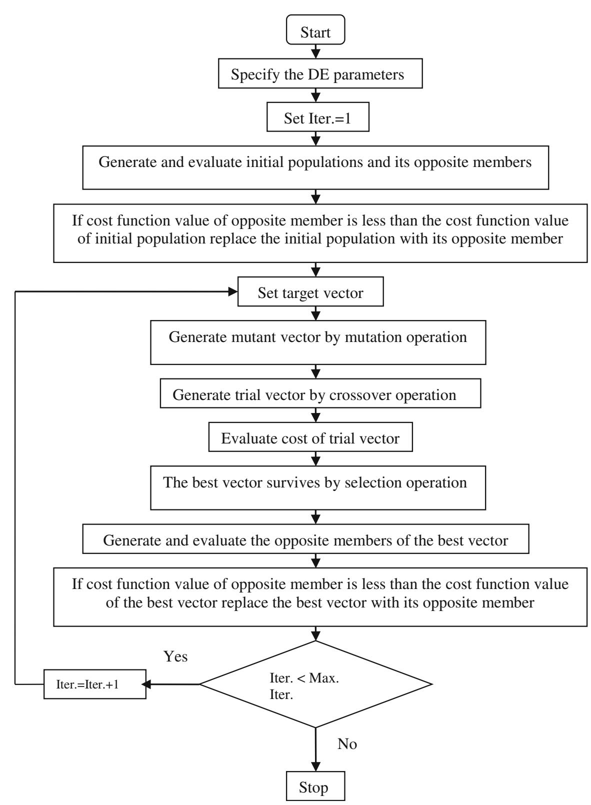

Fig. 2 Flowchart of ODE

Table 1 Best optimum value, the variables corresponding to the best optimum value, average value, worst value and average CPU time for example 1

Method	x^*	$f(x^*)$	Average value	Worst value	CPU time (sec)
ODE	[12.1000, 5.7227]	38.9377	38.9377	38.9377	0.0473
DE	[12.1000, 5.7228]	38.9375	38.9373	38.9371	0.0469

Here, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected 10, 0.3, 1.0 and 50 respectively. The best optimum value, the variables corresponding to the best optimum value, average and worst value and average CPU time among 100 runs of solutions obtained from proposed ODE and DE for example 1 have been shown in Table 1. Figure 3 shows the nature of convergence obtained from ODE and DE for example 1.

Example 2: Consider the minimization problem [31]

$$\min_{x_1,x_2} f(x_1,x_2) = \sum_{i=1}^{5} i\cos[(i+1)x_1+i]\sum_{i=1}^{5} i\cos[(i+1)x_2+i] \tag{27}$$

where $-10 \le x_1 \le 10$ and $-10 \le x_2 \le 10$

This function has 760 local minima, 18 of which are global minima with −186.73. The problem is solved by using ODE. Here, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected 10, 0.3, 1 and 100 respectively for the example under consideration.

To validate the proposed ODE based approach, the same example is solved by using DE.

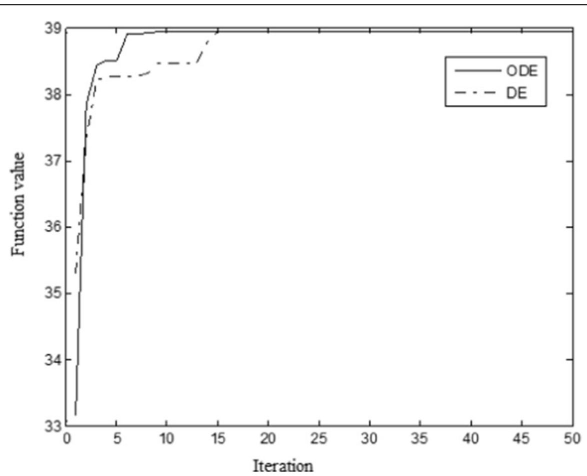

Fig. 3 Convergence characteristic of example 1

Table 2 Best optimum value, the variables corresponding to the best optimum value, average value, worst value and average CPU time for example 2

Method	x^*	$f(x^*)$	Average value	Worst value	CPU time (sec)
ODE	[5.4830, 4.8581]	−186.7309	−186.7309	−186.7309	0.0625
DE	[−7.7084, −7.0834]	−186.7308	−186.7307	−186.7303	0.0781

In case of DE, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected as 10, 0.3, 1.0 and 100 respectively. Table 2 summarizes the best optimum value, the variables corresponding to the best optimum value, average and worst value and average CPU time among 100 runs of solutions obtained from proposed ODE and DE for example 2. Figure 4 depicts the nature of convergence obtained from ODE and DE for example 2.

Figure 4 depicts the nature of convergence obtained from ODE and DE for example 2.

Case study of fixed head hydrothermal system

Test system 1

This system consists of two hydro plants and two thermal plants whose characteristics and load demands are given in Tables 13, 14 and 15 respectively in Appendix 1. Transmission loss formula coefficients are also given in the Appendix 1. Hydro plant data is taken from [32].

The problem is solved by using both the proposed ODE and DE. Here, the population size (N_P), scaling factor (F), crossover rate (C_R) and the maximum iteration number ($N_{\max}$) have been selected as 100, 1.0, 1.0 and 100 respectively for the test system under consideration.

The optimal hydrothermal generation obtained by the proposed ODE and DE are provided in Tables 3 and 4 respectively. The best, average and worst cost and

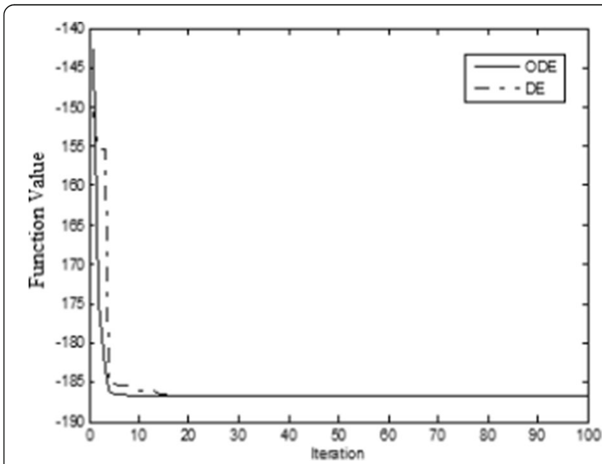

Fig. 4 Convergence characteristic of example 2

Table 3 Results obtained from ODE of test system 1 of fixed head hydrothermal system

Sub-interval	P_{h1} (MW)	P_{h2} (MW)	P_{s1} (MW)	P_{s2} (MW)
1	244.5860	90.7689	179.4953	424.9773
2	307.3581	163.3383	228.7850	570.1572
3	285.4852	139.2931	211.2739	522.5895

Table 4 Results obtained from DE for test system 1 of fixed head hydrothermal system

Sub-interval	P_{h1} (MW)	P_{h2} (MW)	P_{s1} (MW)	P_{s2} (MW)
1	240.3807	85.6583	206.3934	407.6673
2	310.1176	167.5754	206.3934	585.2895
3	286.6845	139.7912	206.3934	525.7479

Table 5 Comparison of performance for Test System 1 of fixed head hydrothermal system

Techniques	Best cost ($)	Average cost ($)	Worst cost ($)	CPU time (s)
ODE	66030.85	66031.68	66032.46	40.31
DE	66060.74	66061.44	66064.14	36.01
AIS [12]	66117	-	-	53.43
PSO [12]	66166	-	-	71.62
EP [12]	66198	-	-	75.48

average CPU time among 100 runs of solutions obtained from proposed ODE and DE method are summarized in Table 5. The cost obtained from artificial immune system (AIS) [14], particle swarm optimization (PSO) [14] and evolutionary programming (EP) [14] are also shown in Table 5. The cost convergence characteristic obtained from proposed ODE and DE is shown in Fig. 5. It is seen from Table 5 that the cost found by using ODE is the lowest among all other methods.

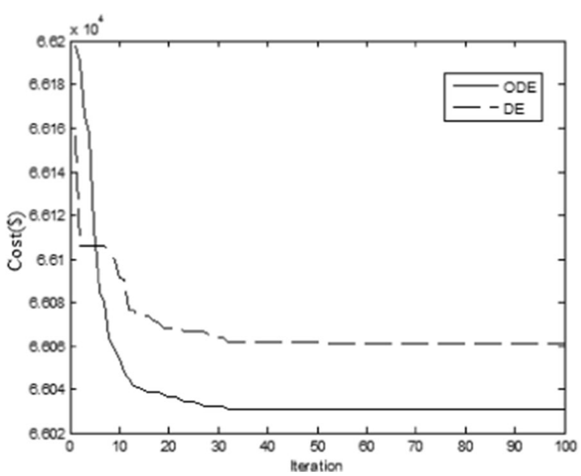

Fig. 5 Cost convergence of test system 1 of fixed head hydrothermal system

Test system 2

This system comprises of two hydro plants and four thermal plants whose characteristics and load demands are given in Tables 16, 17 and 18 respectively in Appendix 2. Transmission loss formula coefficients are also given in the Appendix 2.

The problem is solved by using both the proposed ODE and DE. Here, the population size (N_P), scaling factor (F), crossover rate (C_R) and the maximum iteration number ($N_{\max}$) have been selected as 100, 1.0, 1.0 and 200 respectively for the test system under consideration. The optimal hydrothermal generation obtained by the proposed ODE and DE are provided in Tables 6 and 7 respectively. The best, average and worst cost and average CPU time among 100 runs of solutions obtained from proposed ODE and DE are summarized in Table 8. The cost obtained from artificial immune system (AIS)

Table 6 Results obtained from ODE of test system 2 of fixed head hydrothermal system

Sub-interval	P_{h1} (MW)	P_{h2} (MW)	P_{s1} (MW)	P_{s2} (MW)	P_{s3} (MW)	P_{s4} (MW)
1	172.6478	317.8272	93.6207	174.7438	109.2596	50.3779
2	243.8370	411.3216	124.8716	174.6929	123.6025	50.1150
3	209.7780	351.8750	116.1764	174.7282	120.3243	50.0519
4	249.8641	499.8741	124.8642	174.9127	222.4536	68.0992

Table 7 Results obtained from DE of test system 2 of fixed head hydrothermal system

Sub-interval	P_{h1} (MW)	P_{h2} (MW)	P_{s1} (MW)	P_{s2} (MW)	P_{s3} (MW)	P_{s4} (MW)
1	184.4627	303.6346	88.3611	174.7233	116.2664	50.9170
2	241.0344	419.5791	117.4402	174.8712	124.7407	50.9397
3	201.9931	357.2371	123.3403	173.9739	115.3547	51.0280
4	249.3076	499.1428	124.0676	174.7184	221.4260	71.3501

Table 8 Comparison of performance for Test System 2 of fixed head hydrothermal system

Techniques	Best cost ($)	Average cost ($)	Worst cost ($)	CPU time (s)
ODE	92817.01	92819.81	92822.68	46.09
DE	93107.34	93110.45	93114.07	41.53
AIS [12]	93950	-	-	59.14
PSO [12]	94126	-	-	83.54
EP [12]	94250	-	-	67.82

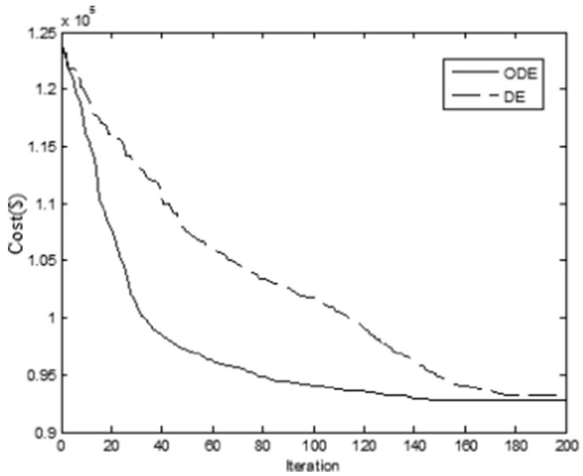

Fig. 6 Cost convergence of test system 2 of fixed head hydrothermal system

[14], particle swarm optimization (PSO) [14] and evolutionary programming (EP) [14] are also shown in Table 8. The cost convergence characteristic obtained from proposed ODE and DE is depicted in Fig. 6. It is seen from Table 8 that the cost found by using ODE is the lowest among all other methods.

Case study of variable head hydrothermal system

Three variable head hydrothermal test systems are considered to inspect and verify the proposed ODE method.

Test system 1

This test system considers a multi-chain cascade of four reservoir hydro plants and an equivalent thermal plant. The entire scheduling period is 1 day and divided into 24 intervals. Here, two cases are considered.

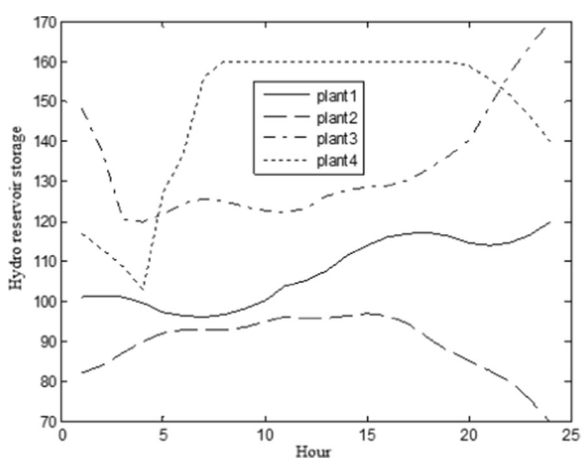

Fig. 7 Hydro reservoir storage volumes for case of test system 1 of variable head hydrothermal system

Table 9 Comparison of performance for case 1 of Test System 1 of variable head hydrothermal system

Techniques	Best cost ($)	Average cost ($)	Worst cost ($)	CPU time (s)
ODE	917199.44	917208.56	917221.37	257.03
DE	918480.03	918494.37	918504.47	256.75
TLBO [16]	922373.39	922462.24	922873.81	-
IPSO [13]	922553.49	-	-	-
MDE [10]	922556.44	-	-	-
IFEP [9]	930129.82	930290.13	930881.92	1033.20
GA [7]	926707.00	-	-	-

Case 1: Here fuel cost is considered as a quadratic function of the power from the composite thermal plant. The detailed parameters for this case come from [7].

The problem is solved by using both the proposed ODE and DE. Here, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected 100, 1, 1 and 300 respectively for this case.

The optimal hourly discharges and hydrothermal generation obtained by the proposed ODE method are provided in Tables 19 and 20 in Appendix 3 respectively. Figure 7 depicts the reservoir storage volumes of four hydro plants obtained from ODE. The best, average and worst cost and average CPU time among 100 runs of solutions obtained from proposed ODE and DE are summarized in Table 9. The cost obtained from modified differential evolution (MDE) [10], improved particle swarm optimization (IPSO) [13], teaching learning based optimization (TLBO) [16], improved fast evolutionary programming (IFEP) [9] and genetic algorithm (GA) [7] methods are also shown in Table 9. The cost convergence characteristic obtained from proposed ODE and DE is shown in Fig. 8. It is seen

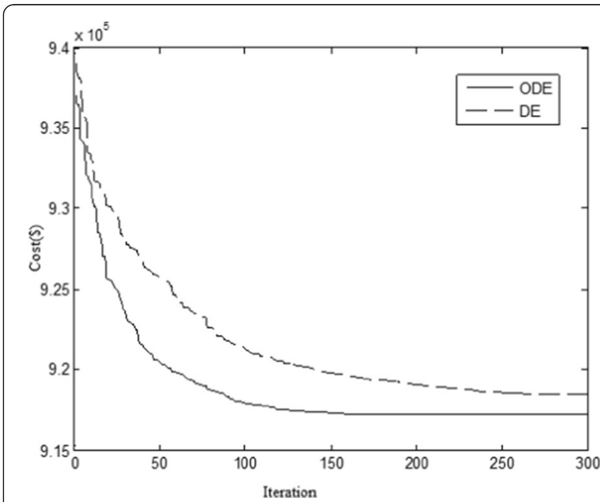

Fig. 8 Cost convergence characteristics for case 1 of the system 1 of variable head hydrothermal system

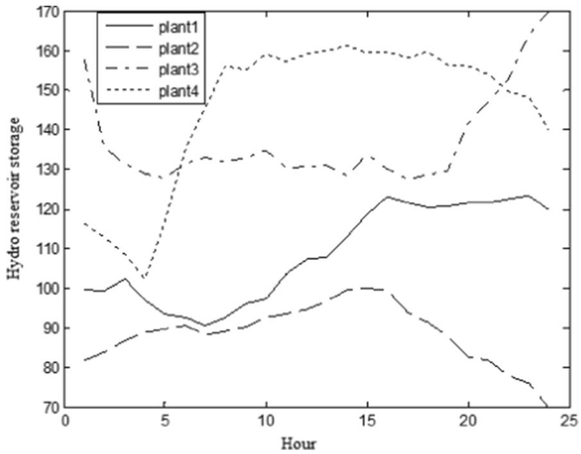

Fig. 9 Hydro reservoir storage volumes for case 2 of test system 1 of variable head hydrothermal system

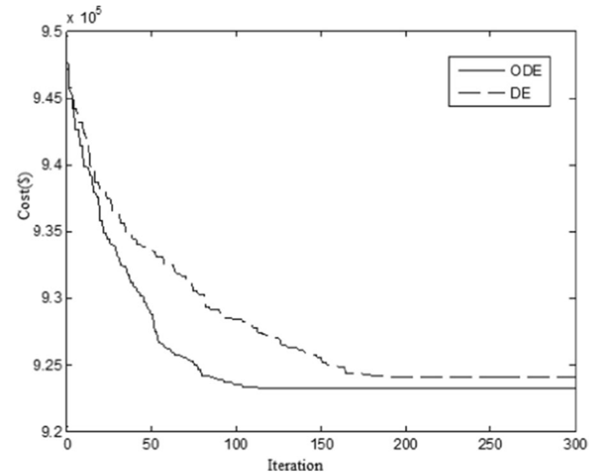

Fig. 10 Cost convergence characteristics for case 2 of the system 1 of variable head hydrothermal system

from Table 9 that the cost found by using ODE is the lowest among all other methods.

Case 2: Here prohibited operating zones of hydro plants and valve point loading of thermal generator are considered. The detailed parameters for this case come from [9].

The problem is solved by using both the proposed ODE and DE. Here, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected 100, 1, 1 and 400 respectively for this case.

The optimal hourly discharges and hydrothermal generation obtained by the proposed ODE method are provided in Tables 21 and 22 respectively in Appendix 3. Figure 9 shows the reservoir storage volumes of four hydro plants obtained from ODE. The best, average and worst cost and average CPU time among 100 runs of solutions obtained from proposed ODE and DE are summarized in Table 10. The cost obtained from improved fast evolutionary programming (IFEP) [9], improved particle swarm optimization (IPSO) [13] and teaching learning based optimization (TLBO) [16] method is also shown in Table 10. The cost convergence characteristic obtained from proposed ODE and DE is shown in Fig. 10. It is seen from Table 10 that the cost found by using ODE is the lowest among all other methods.

Table 10 Comparison of performance for case 2 of test system 1 of variable head hydrothermal system

Techniques	ODE	DE	IFEP [9]	TLBO [16]	IPSO [13]
Best cost ($)	923230.63	924069.73	933949.25	924550.78	925978.84
Average cost ($)	923242.45	924083.56	938508.87	924702.43	-
Worst cost ($)	923255.37	924096.28	942593.02	925149.06	-
CPU time (s)	264.73	258.65	1450.90	-	-

Test system 2

This system considers a multi-chain cascade of four reservoir hydro plants and three thermal plants. The entire scheduling period is 1 day and divided into 24 intervals. The effect of valve point loading is considered. Transmission loss is also considered. The detailed parameters for this case are taken from [10].

The problem is solved by using both the proposed ODE and DE. Here, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected 100, 1, 1 and 300 respectively for this case.

The optimal hourly discharges and hydrothermal generation obtained by the proposed ODE method are provided in Tables 23 and 24 respectively in Appendix 3. Figure 11 shows the reservoir storage volumes of four hydro plants obtained from ODE.

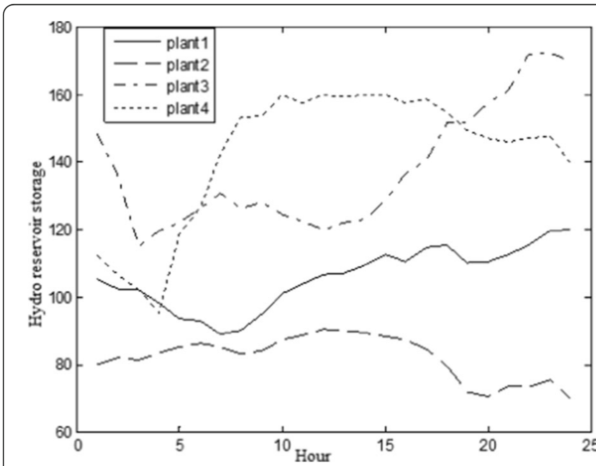

Fig. 11 Hydro reservoir storage volumes of test system 2 of variable head hydrothermal system

Table 11 Comparison of performance of test system 2 of variable head hydrothermal system

Techniques	ODE	DE	MDE [10]	TLBO [3]	CSA [15]
Best cost ($)	42322.23	43068.01	43435.41	42385.88	42440.574
Average cost ($)	42330.53	43079.52	-	42407.23	-
Worst cost ($)	42339.36	43083.05	-	42441.36	-
CPU time (s)	304.05	298.72	-	-	-

Table 12 Comparison of performance for test system 3 of variable head hydrothermal system

Techniques	ODE	DE	DE [12]
Best cost ($)	170452.35	170915.57	170964.15
Average cost ($)	170459.78	170924.41	-
Worst cost ($)	170468.52	170935.28	-
CPU time (s)	472.51	459.92	-

The best, average and worst cost and average CPU time among 100 runs of solutions obtained from proposed ODE and DE are shown in Table 11. The cost obtained from modified differential evolution (MDE) [10], clonal selection algorithm (CSA) [15] and teaching learning based optimization (TLBO) [16] is also shown in Table 11. The cost convergence characteristic obtained from proposed ODE and DE is shown in Fig. 12. It is seen from Table 11 that the cost found by using ODE is the lowest among all other methods.

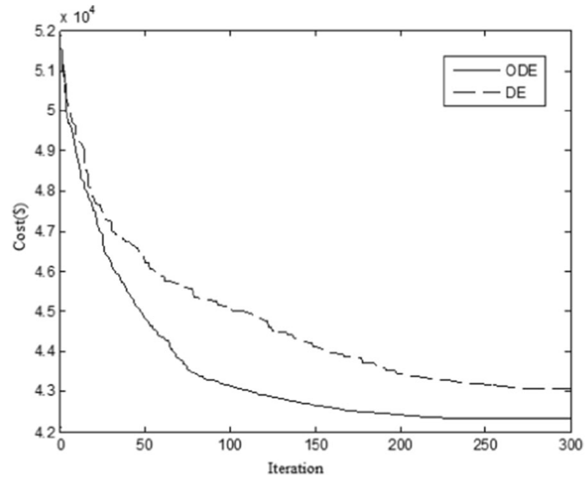

Fig. 12 Cost convergence characteristics of test system 2 of variable head hydrothermal system

Test system 3

This system considers a multi-chain cascade of four reservoir hydro plants and ten thermal plants. The entire scheduling period is 1 day and divided into 24 intervals. The effect of valve point loading is taken into account. Here transmission loss is not considered. The detailed data for this system is taken from [12].

The problem is solved by using both the proposed ODE and DE. Here, the population size (N_P), scaling factor (F), crossover constant (C_R) and maximum iteration number have been selected 100, 1, 1 and 900 respectively for this case.

The optimal hourly discharges and hydrothermal generation obtained by the proposed ODE method are provided in Tables 25 and 26 respectively in Appendix 3. Figure 13 shows the reservoir storage volumes of four hydro plants obtained from ODE. The best, average and worst cost and average CPU time among 100 runs of solutions obtained from proposed

ODE and DE are summarized in Table 12. The cost obtained from differential evolution (DE) [12] method is

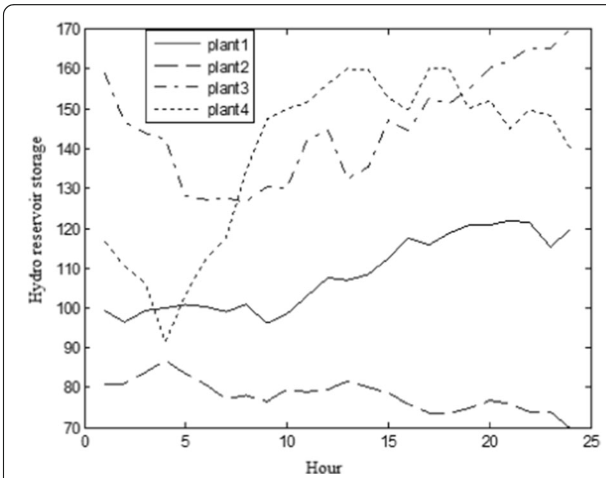

Fig. 13 Hydro reservoir storage volumes of test system 3 of variable head hydrothermal system

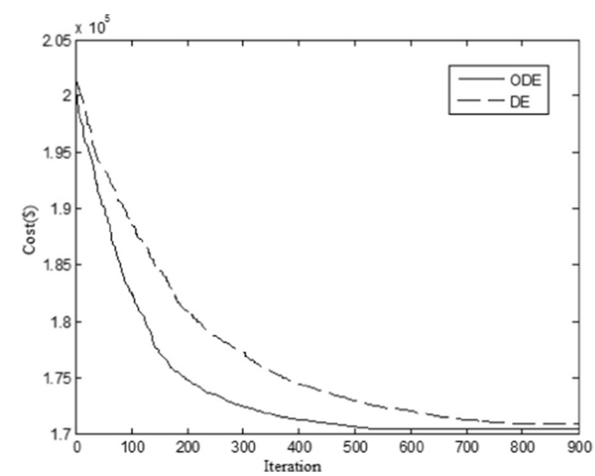

Fig. 14 Cost convergence characteristics for test system 3 of variable head hydrothermal system

also shown in Table 12. The cost convergence characteristic obtained from proposed ODE and DE is shown in Fig. 14. It is seen from Table 12 that the cost found by using ODE is the lowest among all other methods.

It is observed from in Tables 20, 22, 24 and 26 respectively in Appendix 3 that the third hydro unit has no output during some time interval. This is because of the fact that output from a particular hydro unit during a specified time interval depends on the availability of water, reservoir storage volume limit, water transport delay between cascaded reservoirs and on the system configuration as a whole. Depending on the system configuration and constraints for the present problem, this has happened in case of the third hydro unit.

Conclusion

In this paper, opposition-based differential evolution is demonstrated and presented to solve the hydrothermal scheduling problem. The proposed opposition-based differential evolution method has been successfully applied to two test problems, two fixed head hydrothermal test systems and three hydrothermal multi-reservoir cascaded hydroelectric test systems having prohibited operating zones and thermal units with valve point loading. The results have been compared with those obtained by other evolutionary algorithms reported in the literature. It is seen from the comparisons that the proposed opposition-based differential evolution method performs better than other evolutionary algorithms in the literature.

Nomenclature

a_{si}, b_{si}, c_{si}, d_{si}, e_{si}: cost curve coefficients of i th thermal unit

P_{sim}: power output of i th thermal generator during subinterval m

$P_{si}^{\min}$, $P_{si}^{\max}$: lower and upper generation limits for i th thermal unit

t_m: duration of subinterval m.

P_{hjm}: power output of j th hydro unit during subinterval m

P_{Dm}: load demand during subinterval m

P_{Lm}: transmission loss during subinterval m

B_{lr}: loss formula coefficients.

a_{0hj}, a_{1hj}, and a_{2hj}: coefficients for water discharge rate function of j th hydro generator

W_{hj}: prespecified volume of water available for generation by j th hydro unit during the scheduling period.

$P_{hj}^{\min}$, $P_{hj}^{\max}$: lower and upper generation limits for j th hydro unit

P_{sit}: output power of i th thermal unit at time t

P_{Dt}: load demand at time t

P_{Lt}: transmission loss at time t

P_{hjt}: output power of j th hydro unit at time t

C_{1j}, C_{2j}, C_{3j}, C_{4j}, C_{5j}, C_{6j}: power generation coefficients of j th hydro unit

Q_{hjt}: water discharge rate of j th reservoir at time t

V_{hjt}: storage volume of j th reservoir at time t

$Q_{hj}^{\min}$, $Q_{hj}^{\max}$: minimum and maximum water discharge rate of j th reservoir

$Q_{hj,k}^{L}$, $Q_{hj,k}^{U}$: lower and upper bounds of k th prohibited zones of hydro unit j

$V_{hj}^{\min}$, $V_{hj}^{\max}$: minimum and maximum storage volume of j th reservoir

I_{hjt}: inflow rate of j th reservoir at time t

R_{uj}: number of upstream units directly above j th hydro plant

S_{hjt}: spillage of j th reservoir at time t

τ_{lj}: water transport delay from reservoir l to j

t, T: time index and scheduling period

N_s: number of thermal generating units

N_h: number of hydro generating units

n_j: number of prohibited zones for hydro unit j

k: index of prohibited zones of a hydro unit

Appendix 1

Table 13 Hydro system data of test system 1

Unit	a_{0h}	a_{1h}	a_{2h}	W_h	$P_h^{\min}$	$P_h^{\max}$
	MCF/h	MCF/MWh	MCF/(MW)2 h	MCF	MW	MW
1	1.980	0.306	0.000216	2500	0	400
2	0.936	0.612	0.000360	2100	0	300

Table 14 Thermal generator data of test system 1

Unit	$P_s^{\min}$	$P_s^{\max}$	a_s	b_s	c_s	d_s	e_s
	MW	MW	$/h	$/MWh	$/(MW)2 h	$/h	1/MW
1	50	300	25	3.2	0.0025	0	0
2	50	700	30	3.4	0.0008	0	0

Table 15 Load demands of test system 1

Sub-interval	Duration (hr)	PD (MW)
1	8	900
2	8	1200
3	8	1100

The transmission loss formula coefficients of test system 1 are

$$\mathrm{B} = \begin{bmatrix} 0.000015 & 0.000140 & 0.000010 & 0.000015 \\ 0.000010 & 0.000060 & 0.000010 & 0.000013 \\ 0.000015 & 0.000010 & 0.000068 & 0.000065 \\ 0.000015 & 0.000013 & 0.000065 & 0.000070 \end{bmatrix}$$

Appendix 2

Table 16 Hydro system data of test system 2

Unit	a_{0h}	a_{1h}	a_{2h}	W_h	P_h^{min}	P_h^{max}
	acre-ft/h	acre-ft/MWh	acre-ft/(MW)2 h	acre-ft	MW	MW
1	260	8.5	0.00986	125000	0	250
2	250	9.8	0.01140	286000	0	500

Table 17 Thermal generator data of test system 2

Unit	P_s^{min}	P_s^{max}	a_s	b_s	c_s	d_s	e_s
	MW	MW	$/h	$/MWh	$/(MW)2 h	$/h	rad/MW
3	20	125	10	3.25	0.0083	12	0.0450
4	30	175	10	2.00	0.0037	18	0.0370
5	40	250	20	1.75	0.0175	16	0.0380
6	50	300	20	1.00	0.0625	14	0.0400

Table 18 Load demands of test system 2

Sub-interval	Duration (hr)	PD (MW)
1	12	900
2	12	1100
3	12	1000
4	12	1200

The transmission loss formula coefficients are of test system 2

$$B = \begin{bmatrix} 0.000049 & 0.000014 & 0.000015 & 0.000015 & 0.000020 & 0.000017 \\ 0.000014 & 0.000045 & 0.000016 & 0.000020 & 0.000018 & 0.000015 \\ 0.000015 & 0.000016 & 0.000039 & 0.000010 & 0.000012 & 0.000012 \\ 0.000015 & 0.000020 & 0.000010 & 0.000040 & 0.000014 & 0.000010 \\ 0.000020 & 0.000018 & 0.000012 & 0.000014 & 0.000035 & 0.000011 \\ 0.000017 & 0.000015 & 0.000012 & 0.000010 & 0.000011 & 0.000036 \end{bmatrix} \text{per MW}$$

Appendix 3

Table 19 Optimal Hydro Discharge ($\times 10^4 m^3$) for case 1 of Test System 1 of variable head hydrothermal system

Hour	Q_{h1}	Q_{h2}	Q_{h3}	Q_{h4}
1	8.7861	6.0009	30.0000	6.0000
2	8.6477	6.0001	18.5747	6.0000
3	8.5682	6.0000	29.9998	6.0000
4	8.3775	6.0006	17.3534	6.0008
5	8.1550	6.0000	15.4229	6.0005
6	8.0533	6.0030	15.9130	7.9993
7	8.1591	6.0910	15.9792	11.1179
8	8.4589	6.8847	16.5977	13.6690
9	8.6193	7.4527	16.4652	15.3635
10	8.7715	7.6903	16.5940	16.1257
11	8.5801	7.7683	17.1467	15.7670
12	8.6525	8.1049	16.8463	16.5977
13	8.5011	8.2039	17.4470	16.4653
14	8.3269	8.3350	17.8223	16.5934
15	8.2464	8.4235	18.7109	17.1544
16	8.0697	8.7110	18.4832	16.8390
17	8.0004	9.0106	16.9627	17.4464
18	7.8467	9.4610	15.9095	17.8224
19	7.8246	10.1045	14.5644	18.8539
20	7.7368	10.6701	13.8283	19.6055
21	7.5925	11.2530	11.0169	19.9997
22	7.3682	11.7971	11.5735	19.9999
23	6.9536	12.6091	12.0326	19.9999
24	6.7040	13.4245	12.5674	19.9998

Table 20 Optimal Hydrothermal generation (MW) for case 1 of test system 1 of variable head hydrothermal system

Hour	P_{h1}	P_{h2}	P_{h3}	P_{h4}	P_s
1	79.7973	49.0061	0	131.8801	1109.32
2	79.3927	50.1639	43.5292	129.0270	1087.89
3	79.0387	51.2957	0	125.7437	1103.92
4	77.7373	52.9380	37.4242	121.6365	1000.26
5	75.9674	54.4995	42.2628	115.8283	1001.44
6	74.6619	55.5248	42.0011	163.8960	1073.92
7	74.9610	56.6535	42.7802	209.7731	1265.83
8	76.6787	62.1650	41.6644	252.8746	1566.62
9	77.7838	65.9683	41.8104	271.8340	1782.60
10	79.1114	67.7564	40.9661	278.4111	1853.75
11	78.7489	68.9033	38.9557	275.1930	1768.19
12	80.1994	71.5905	39.5975	282.2694	1836.34
13	79.6781	72.1369	38.3010	281.2003	1758.68
14	79.2573	72.8195	38.1722	282.2342	1727.52
15	79.5884	73.6734	35.5391	286.6439	1654.55
16	78.9796	75.6289	36.7765	284.1818	1594.43
17	78.8516	76.9618	41.8592	288.8606	1643.46
18	77.9593	78.3512	45.1334	291.6388	1646.92
19	77.8291	79.6915	48.4354	298.8079	1735.23
20	77.0919	80.4924	50.2710	303.4720	1768.67
21	75.8005	81.3147	51.4605	304.7025	1726.72
22	74.1001	81.9619	53.9109	301.5554	1608.47
23	71.1238	82.8437	56.0420	297.2275	1342.77
24	69.4655	81.8843	57.7491	291.3201	1089.58

Table 21 Optimal Hydro Discharge ($\times 10^4 m^3$) for case 2 of test system 1 of variable head hydrothermal system

Hour	Q_{h1}	Q_{h2}	Q_{h3}	Q_{h4}
1	10.1845	6.1121	20.5536	6.3438
2	9.3545	6.0000	29.9857	6.0059
3	5.0934	6.0672	18.8188	6.0081
4	12.3025	6.9922	19.7814	6.0011
5	9.4396	6.9832	15.2970	6.3376
6	7.8835	6.3622	18.4255	11.1545
7	10.2721	8.2105	18.0212	8.7499
8	6.7694	6.0283	17.9212	9.3215
9	6.6014	6.9949	16.6465	15.9994
10	9.8394	6.6298	14.1732	14.6373
11	5.8365	8.0881	17.9684	19.8695
12	6.2467	6.7252	18.3894	15.9965
13	10.4311	6.0065	16.4035	15.9976
14	6.7118	6.0342	19.8262	13.0358
15	5.2117	8.9019	14.7661	19.6512
16	5.8669	8.0785	18.5218	18.0045
17	10.3436	13.0473	15.8221	18.0241
18	9.0289	8.2601	15.6486	18.1861
19	6.8068	10.6257	18.4059	18.1376
20	5.0351	13.1212	10.7805	18.6221
21	7.2673	9.9088	11.9574	18.0174
22	7.0480	12.8178	11.9622	20.0000
23	7.9655	10.0050	10.1140	19.8378
24	13.2600	13.2228	11.6386	19.6248

Table 22 Optimal Hydrothermal generation (MW) for case 2 of test system 1 of variable head hydrothermal system

Hour	P_{h1}	P_{h2}	P_{h3}	P_{h4}	P_s
1	86.8344	49.7921	42.4872	136.4915	1054.39
2	82.7927	50.0996	0	128.7959	1128.31
3	53.2023	51.7131	38.2060	125.5285	1091.35
4	95.3572	59.6889	32.3296	121.2998	981.32
5	82.2469	60.718	45.8186	119.8371	981.38
6	72.3219	57.0068	36.5382	189.8853	1054.25
7	84.3272	69.3217	39.5418	180.8054	1276.01
8	63.8671	53.8113	40.4187	196.2406	1645.66
9	63.4079	61.0171	43.8752	273.675	1798.02
10	83.8010	59.0764	48.8933	261.1712	1867.06
11	58.8915	69.7213	40.9075	304.3079	1756.17
12	63.6159	61.3895	37.7617	274.8044	1872.42

Table 22 Optimal Hydrothermal generation (MW) for case 2 of test system 1 of variable head hydrothermal system *(Continued)*

13	90.3650	56.9064	44.1043	276.5894	1762.03
14	68.1055	58.0340	32.0219	249.5939	1792.24
15	56.4505	78.0365	46.5805	292.2578	1603.17
17	93.8183	96.3354	45.1644	292.7708	1601.91
18	86.3257	71.3305	44.6121	292.334	1645.39
19	70.6874	82.4647	36.9713	293.5957	1756.28
20	55.3803	89.5719	48.3725	293.5926	1793.08
21	74.3523	73.9104	52.4342	289.3438	1749.96
22	72.6529	84.7098	53.7410	299.7932	1609.10
23	79.4951	71.3102	53.2242	294.2187	1351.75
24	104.9608	81.7733	57.2461	291.4408	1054.58

Table 23 Optimal Hydro Discharge ($\times 10^4 m^3$) of test system 2 of variable head hydrothermal system

Hour	Q_{h1}	Q_{h2}	Q_{h3}	Q_{h4}
1	5.0000	8.1694	29.9825	10.6846
2	11.8249	6.0349	20.3834	8.1109
3	8.2756	9.3968	29.9993	6.0699
4	10.6764	7.1839	17.4356	6.5270
5	10.7913	6.1217	14.9166	7.0655
6	7.5122	6.0114	19.9168	12.2241
7	11.8929	7.1014	16.4236	14.2319
8	8.0364	8.9342	19.9639	6.3860
9	5.0000	7.0265	17.2913	14.8253
10	5.2012	6.0000	19.6801	13.3341
11	9.0382	7.4124	16.8647	18.8811
12	7.1895	6.0830	16.7021	17.6400
13	10.7560	8.4874	17.0601	18.0055
14	9.6444	9.6666	16.3546	18.8809
15	7.5333	10.1478	14.5476	16.8217
16	12.2331	9.0725	12.3182	19.4624
17	5.0001	9.8397	14.7639	16.0024
18	6.9996	10.8825	13.7793	20.0000
19	12.3816	14.8071	14.5850	20.0000
20	5.7002	9.2668	12.3534	14.4891
21	5.0013	6.0008	21.3704	15.8796
22	5.0078	9.1880	11.7756	12.9617
23	5.0002	6.0045	15.2021	13.6869
24	9.3038	13.1606	12.9722	19.9519

Table 24 Optimal Hydrothermal generation (MW) of test system 2 of variable head hydrothermal system

Hour	P_{h1}	P_{h2}	P_{h3}	P_{h4}	P_{s1}	P_{s2}	P_{s3}
1	52.5001	62.9911	0	188.4124	20.0000	40.0470	409.0353
2	94.9645	49.1472	36.0943	151.4483	20.0001	294.7080	139.9935
3	77.4693	70.7940	0	120.2516	174.9999	40.0626	229.7881
4	89.6264	57.8274	34.8294	121.7495	174.9999	40.0144	140.0427
5	88.6817	51.7208	43.1194	121.7213	20.0713	209.8746	139.7717
6	69.9013	51.9834	27.7926	202.2525	20.0027	294.7478	139.7384
7	90.0410	59.9969	42.2954	229.2022	102.8131	294.9635	140.0704
8	71.5381	70.3084	31.2740	155.4249	175.0000	294.7975	229.5029
9	49.9772	57.7790	39.9087	261.0326	174.9942	294.7360	229.4873
10	53.0657	51.3073	31.4504	247.0815	102.6427	294.7893	319.3190
11	81.4462	62.5231	40.4364	298.9633	20.0014	294.7375	319.3074
12	70.7288	54.3802	40.1454	287.9098	102.6722	294.7042	319.2878
13	91.5678	70.9632	38.1093	292.8062	20.0158	294.6822	319.3190
14	86.4941	77.0705	40.8922	298.1207	102.6981	294.7381	139.8472
15	74.3655	79.0482	45.0727	83.9845	102.6488	294.7742	139.7885
16	98.9002	72.9424	48.6283	302.6975	20.0008	298.7904	229.5013
17	54.2296	76.2645	49.2103	274.8114	174.9981	294.7637	139.6912
18	71.4333	79.2985	51.6777	304.2234	102.6951	294.7382	229.7389
19	100.2199	87.8464	53.8828	300.4273	102.7774	294.7563	140.0848
20	60.3478	63.3576	55.0328	254.1731	20.0000	294.7757	319.3300
21	54.2311	42.9424	34.5069	264.0216	175.0000	40.0042	319.0230
22	54.5201	64.1698	56.8181	236.6617	20.0000	294.7116	139.6709
23	54.7321	44.8122	58.1308	244.4422	20.0024	294.6434	139.7895
24	87.5753	80.9892	59.3598	292.6200	20.0004	125.0043	139.8794

Table 25 Optimal Hydro Discharge ($\times 10^4 m^3$) of test system 3 of variable head hydrothermal system

Hour	Q_{h1}	Q_{h2}	Q_{h3}	Q_{h4}
1	10.5900	7.2207	19.4370	6.0254
2	12.0523	7.7304	20.2455	8.5457
3	5.0001	6.0184	17.4557	6.0000
4	6.4478	6.3207	22.6585	14.8061
5	5.0000	11.1350	29.9287	7.6698
6	7.6269	9.9408	17.6070	10.8973
7	9.2146	9.5815	13.9492	12.4732
8	7.1216	6.0000	21.4589	6.0044
9	14.7220	9.4742	16.3758	16.8335
10	8.7003	6.0001	18.0804	15.0361
11	7.6528	9.7120	10.0203	12.3636
12	5.4338	7.2947	17.1649	16.8305
13	11.5460	6.0053	30.0000	12.6269
14	10.5001	10.4945	15.3613	18.1704
15	6.9555	10.5776	10.0003	17.1377
16	5.0000	10.6310	21.2541	19.9868
17	10.5398	9.0909	11.1185	19.9873
18	5.1753	6.0028	19.1245	15.2733
19	5.0000	6.0000	18.4536	19.9871
20	5.8448	6.0003	10.0100	19.2115
21	6.0854	9.8456	11.2876	17.9333
22	8.5236	11.1071	10.4763	14.4865
23	14.9775	7.8296	13.2974	19.9983
24	5.2899	11.9870	13.3435	18.2195

Table 26 Optimal Hydrothermal generation (MW) for test system 3 of variable head hydrothermal system

Hour	P_{h1}	P_{h2}	P_{h3}	P_{h4}	P_{s1}	P_{s2}	P_{s3}	P_{s4}	P_{s5}	P_{s6}	P_{s7}	P_{s8}	P_{s9}	P_{s10}
1	88.5676	57.2201	47.0843	132.2239	139.7462	199.5787	94.9554	119.9507	274.6901	139.7256	45.0047	134.5020	98.1889	178.5618
2	93.4214	60.8649	40.3432	161.1911	50.0010	350.6038	20.4259	20.0004	224.3871	239.4088	281.9108	85.1760	25.0195	127.2461
3	51.6806	49.7455	46.6711	123.3513	229.2817	124.0582	20.0000	119.8332	274.3626	89.8035	163.1392	134.3029	97.0795	176.6904
4	64.1960	53.5788	22.2002	210.9095	229.2333	124.6531	20.0501	119.5701	224.5807	139.8210	104.1674	134.6485	25.0013	177.3898
5	52.4810	81.8434	0	124.6475	140.1426	199.9715	95.3239	120.0332	175.6493	40.2031	222.6977	135.1454	103.3599	178.5014
6	72.9780	74.5900	39.6390	172.7592	228.4289	199.5966	20.1401	69.3630	174.6341	289.5033	222.8367	134.7805	25.0000	75.7505
7	82.2228	70.9868	47.2010	198.3559	318.9251	199.5684	94.6931	119.9201	174.6363	139.7735	45.0001	234.5971	97.7873	126.3325
8	69.0212	47.2262	22.2530	129.7342	229.4300	422.7929	95.4989	119.5967	273.9787	189.4554	102.8809	84.9181	97.3506	125.8631
9	98.8275	68.7802	42.6653	258.0113	319.3601	423.9171	20.3141	69.8048	25.0100	139.9045	163.4803	184.4494	98.0940	177.3813
10	78.0403	46.9347	38.9243	257.2106	319.8205	274.7959	95.2935	69.9514	224.6139	189.1829	163.6077	35.0485	159.9696	126.6063
11	72.4527	70.9485	47.7153	234.1379	319.2929	124.7317	94.8140	120.2192	224.5801	139.5221	341.3076	35.0054	98.2030	177.0697
12	56.8438	56.9932	45.9128	275.8907	230.0333	50.7516	20.1698	119.5267	379.1170	289.1176	104.2716	184.7861	159.9982	176.5876
13	94.8980	48.7840	0	242.2134	139.8324	274.4415	95.1403	69.8449	469.9935	89.6793	163.3926	84.8894	159.9984	176.8925
14	90.5802	75.8706	46.9657	293.9404	229.3965	274.4531	94.6048	20.0137	273.2281	139.8295	281.7942	35.0008	98.4802	75.8422
15	70.0608	75.2619	49.0555	286.2452	50.0001	424.0944	94.7114	129.9991	174.5623	40.0000	45.0649	284.1163	159.9975	126.8307
16	54.4620	74.4441	31.4282	298.2087	229.3469	199.3362	129.9942	70.0416	25.0000	188.9987	400.0086	134.4338	97.6077	126.6892
17	93.6431	65.3782	52.5869	295.1849	50.0097	273.9768	94.4723	69.8887	74.8041	289.2354	281.7366	184.3100	98.3925	126.3805
18	56.3901	45.1903	43.0147	270.9021	229.4754	423.8369	94.6491	69.8844	174.6510	189.6651	163.6530	184.4627	98.1968	76.0285
19	54.9594	45.1688	44.9555	305.7963	454.3275	274.4155	20.3014	69.5663	74.3859	239.4500	104.2637	184.7943	159.9911	37.6244
20	62.7075	45.8064	53.4968	290.7342	50.0001	274.6687	94.9396	119.5349	324.1705	90.0924	222.8150	134.4347	159.9665	126.6326
21	64.7903	69.7331	56.2031	284.4660	229.4512	349.3310	20.0004	20.0019	273.7897	139.5154	45.0000	184.7127	97.3568	75.6484
22	83.1783	74.6517	55.5310	250.3066	319.5464	349.2805	20.0039	119.7490	75.5053	40.0554	163.3451	84.7404	98.4503	125.6563
23	107.4231	57.0151	58.0351	295.2245	229.3402	199.7585	95.1532	119.6982	75.1793	140.3943	163.4162	84.8138	98.1993	126.3492
24	57.3775	76.5220	57.9827	282.6004	319.2834	274.0316	94.4750	69.6811	25.0002	139.5081	45.0000	134.7093	98.0004	125.8284

Authors' contributions
JKP makes substantial contributions to conception, design, acquisition of data, analysis and interpretation of data. JKP drafted the article and revising it thoroughly for preparation of the manuscript for the esteemed journal. Also he did the simulation part by using different test data for two different test systems. As a corresponding author he takes the primary responsibility for communication of the journal during the manuscript submission, peer review, publication process, and typically ensures that all the journal's administrative requirements, such as providing details of authorship. JKP will be available throughout the submission and peer review process to respond to editorial queries in a timely manner. Also he will be available after publication to respond to critiques of the work and cooperate with any requests from the journal for data or additional information should be answered about the paper arise after publication. JKP also agrees to be accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved. MB participated in the peer review process of the manuscript and involved in the test data preparation. She reviewed the manuscript thoroughly. She also involved in preparation for testing of the test systems. DPD participated in the peer review process of the manuscript and to compare the performance of the proposed method with that of other evolutionary methods. He also involved in review the manuscript. All authors read and approved the final manuscript.

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Competing interests
The authors declare that they have no competing interests.

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